ANSWERS

Chapter 1

Exercise 1a Stemplots (page 8)

NOTE: There are alternative formats

1. (a)

50	2	
55	2 2 4	
60	1 3 4 4	
65	0 2 2 3 3 3	
70	0 1 1 2 3 4 4	
75	0 1 1 2 4 4	
80	1 3	
85	1	

Key: 85 | 1 means 86

(b) 68 kg

2.

3	2 2 2 2	
3	0 1 1 1	
2	9 9	
2	6 6 6 6 7 7 7	
2	4 5 5	

Key: 2 | 7 means 27

3.

1	4 4 4 4	
1	7 7 7 7 7 7	
1	8	
2	0 0 0 0 1 1 1 1	
2	2 2 3 3 3 3	
2	4 4 4	
2	6 6	

Key: 2 | 1 means 0.21 seconds

4.

3	9	
4		
5	3 4 5 5	
6	1 1 5 7 8	
7	0 0 1 3 4 5 6 6 8 9	
8	0 1 2 2 4 8	
9	2 6	
10	0 1	

Key: 5 | 3 means 5.3 cm

5.

12	5 9	
11	1 3 6	
10	4	
9	7 8	
8	3 4	
7	0 3 5 5 6 8	
6	1 2 8	
5	6 6 8	
4	3 8	
3		
2	4 6	
1	6	
0	0 2 6 8	

Key: 7 | 3 means 7.3 hours

6. (a) 7.4 hours, 0.5 hrs
(b) 0.074 g, 0.005 g

7. (a)

Before		After
8	4	
7 3 1 1 0	5	
9 9 6 6 4	6	9
9 5 3 3 0 0	7	0 5 5 7 7
1	8	0 0 1 4 4 6
3 3 3 3 1 0 0	9	5 6 7
5 5	10	4 4 4 6 8 9
1 1 0	11	7
	12	5
	13	0 0 1 7 7
	14	3 5

Key: 9 | 7 means 79

Key: 8 | 4 means 84

Rate much faster after exercise

(b)

School A		School B
9 8 7 5 3 3	2	3 5 9
9 9 9 7 7 7 4 3 3 1 1	3	4 6 6 8 8
8 8 8 8 6 6 5 5 5 0 0	4	0 1 2 2 3 4 5 5 6 7 7 9
9 4 4 3 3 1 1	5	0 0 2 2 4 4 6 6 6 7 8 8 9 9 9
1	6	0

Key: 9 | 5 means 59

Key: 5 | 9 means 59

Older teaching staff in School B

(c)

Boys		Girls
	2	4 5 5
3 3 3 2 2	2	2 2 2 2 2 2
1 0	2	1 1
9 9 8 8	1	8 8 9 9 9
6 6 6 6	1	6 6 7 7
5 5 4	1	
	1	
1	1	
9	0	

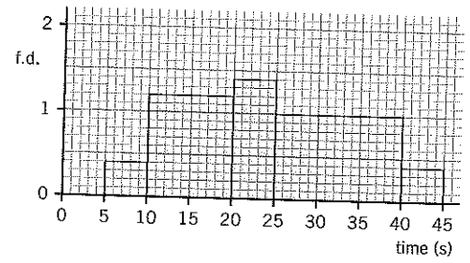
Key: 8 | 1 means 0.18 s

Key: 1 | 8 means 0.18 s

Boys have faster reaction time.
Girls' reaction times more consistent.

Exercise 1b Histograms and frequency polygons (page 21)

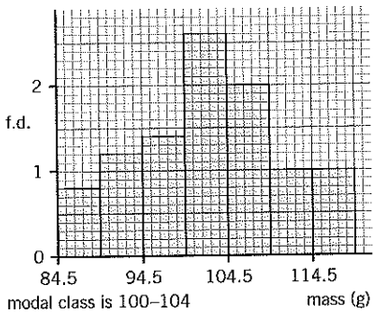
1. Boundary points 5, 10, 20, 25, 40, 45
f.d. 0.4, 1.2, 1.4, 1, 0.4



. (a)

Mass (g)	Frequency	f.d.
85–89	4	0.8
90–94	6	1.2
95–99	7	1.4
100–104	13	2.6
105–109	10	2
110–114	5	1
115–119	5	1

(b)



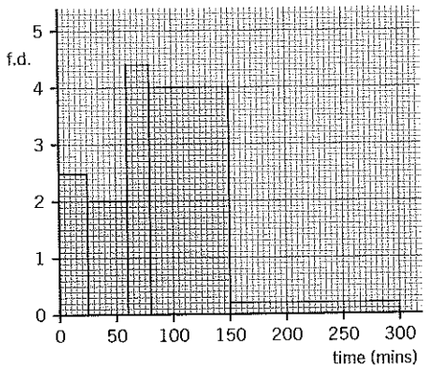
(c)

8	6 6 7 8
9	2 2 2 2 3 3
9	5 6 6 7 8 9 9
10	0 0 0 1 1 1 1 1 2 2 3 3 4
10	5 5 5 6 6 7 7 8 8 9
11	0 1 3 3 4
11	6 6 7 8 8

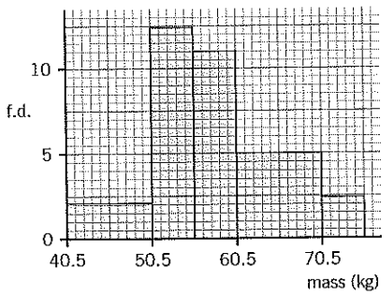
Key: 10 | 3 means 103

mode = 101 g

3. Boundary points 0, 25, 60, 80, 150, 300
f.d. 2.48, 2, 4.4, 4, 0.2



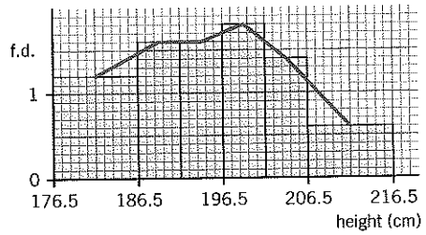
4. Boundary points 40.5, 50.5, 55.5, 60.5, 70.5, 75.5
f.d. 2.1, 12.4, 11, 5, 2.4



5.

Speed	0–	20–	24–	30–	32–	38–	48–	60–
frequency	20	24	24	16	12	10	6	0

6. Boundary points 176.5, 186.5, 191.5, 196.5, 201.5, 206.5, 216.5
f.d. 1.2, 1.6, 1.6, 1.8, 1.4, 0.6



7. Plot polygon at (0.75, 2), (2.25, $4\frac{2}{3}$), (4.5, $7\frac{1}{3}$), (9, $3\frac{1}{3}$), (13.5, 2), (18, 1).

8.

Number of occurrences of e	Frequency	Width	f.d.
0–2	1	3	$\frac{1}{3}$
3–5	5	3	$1\frac{2}{3}$
6–8	6	3	2
9–11	3	3	1
12–14	5	3	$1\frac{2}{3}$
15–17	4	3	$1\frac{1}{3}$

Plot boundaries at –0.5, 2.5, 5.5, 8.5, 11.5, 14.5, 17.5
or at 0, 3, 6, 9, 12, 15, 18

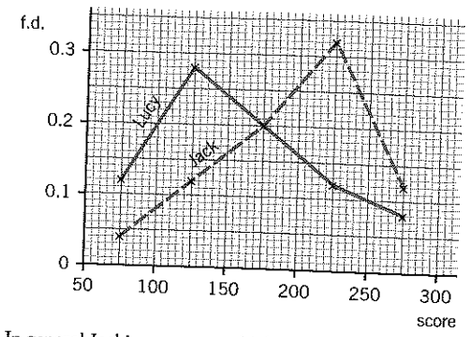
9. Plot polygon at (18, 17.5), (22.5, 94), (27.5, 107), (32.5, 56), (40, 11.8).
Modal class 25–30, skewed with a tail to the right.
(Other answers possible)

10. Boundary points –0.5, 9.5, 14.5, 19.5, 29.5, 39.5, 59.5 (say)
or 0, 10, 15, 20, 30, 40, 60 (say)
f.d. 0.5, 1.6, 6.4, 4.1, 1.6, (0.1).

11. Boundary points 9.5, 29.5, 39.5, 49.5, 59.5, 64.5, 69.5, 84.5
or 10, 30, 40, 50, 60, 65, 70, 85
or 9, 29, 39, 49, 59, 64, 69, 84
f.d. 1.1, 1.8, 2.2, 2.4, 2.8, 2.4, 1.6

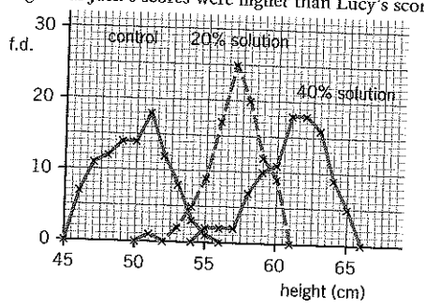
12. 6, 8, 8, 6, 4, 10

13. Take boundary points 50, 100, 150, 200, 250, 300
Lucy: Plot (75, 0.12), (125, 0.28), (175, 0.2), (225, 0.12), (275, 0.08)
Jack: Plot (75, 0.04), (125, 0.12), (175, 0.2), (225, 0.32), (275, 0.12)



In general Jack's scores were higher than Lucy's scores.

14.



The maize seedlings showed a tendency to grow taller with the stronger solution.

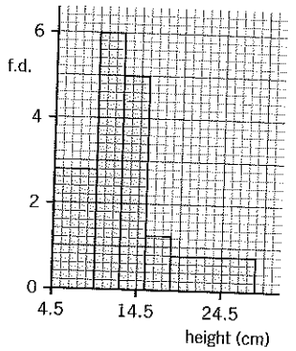
15. (a) a = 20, b = 26, c = 12
(b) 88

Exercise 1c Pie charts (page 26)

1. (a) 154°, 26°, 64°, 116°
(c) 5.51 cm
2. 208°, 46°, 38°, 36°, 32°; 5.25 cm
3. 66°, 156°, 24°, 42°, 72°; 5.5 cm, 6 cm; 50°
4. (a) £120 000 (b) 68 000 (c) 90°, 27°, 9°, 30°; 7.5 cm
5. (a) 42 (b) 40° (c) 91; 420, 30.0 cm
6. (a) 86°, 38°, 32°, 20°, 168°, 16° (b) 5.5 cm
7. (a) £2000, £8000 (b) £400 (c) 27° (d) 80°
8. 28.8°, 72°, 115.2°, 144°; 180
9. (a) £4500 (b) 1550, 1650 (c) 132°, 24°; 8 cm

Exercise 1d The mean (page 34)

1. (a) 9.7 (b) 154.8 (c) 51.375 (d) $1775\frac{5}{7}$
(e) 0.908 (3 s.f.) (f) 4 (g) 29.54 (h) 122.82
2. 49.3
3. 45 (2 s.f.)
4. (a) Boundary points 0, 5, 10, 15, 20, 40
f.d. 2.4, 7.6, 8.4, 4, 0.4
(b) £11.92
5. Boundary points 0, 15, 30, 50, 70, 100
f.d. 3.6, 5.2, 6, 4.4, 2
43.35 years
6. 21.4 cm
7. (a) There should not be gaps between the bars. Heights should be adjusted so that area ∝ frequency
(b) Boundary points 4.5, 9.5, 12.5, 15.5, 18.5, 28.5
f.d. 2.8, 6, 5, $1\frac{1}{3}$, 0.8



(c) 12.9 m (3 s.f.)

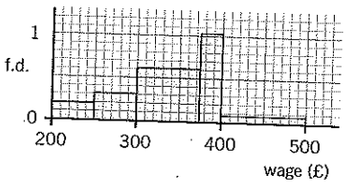
8. (a) Boundary points 9.5, 19.5, 24.5, 29.5, 30.5, 34.5, 39.5, 59.5
f.d. 2, 4, 3, 14, 4, 2, 0.5
(b) 28 seconds.

Exercise 1e Weighted means (page 36)

1. 10.4
2. Class teacher, 1.65%
3. 40.6
4. 4
5. 5, 65.8

Exercise 1f Mean and standard deviation (page 44)

1. (a) 5, 2 (b) 8.5, 1.80 (c) 18.8, 6.46 (d) $10\frac{5}{6}$, 4.10
(e) 3.42, 1.91 (f) 205, 3.16
2. (a) f.d. 0.2, 0.32, 0.625, 1.04, 0.08



(b) £338.25, £59.60
3. 69.3, 1.7
4. 115.8, 7.58
5. 16.6 seconds, 2.63 seconds
6. 6.8, 1.11
7. (a) 2 min 38 sec, 1 min 54 sec
(b) Histogram f.d. 6, 10, 15, 2.5, 0.8
Frequency polygon: plot (0.5, 6), (1.5, 10), (2.5, 15), (4, 2.5), (7.5, 0.8)
8. 29, 5.9
9. 5.10
10. 5
11. (a) 10 (b) 11.7
12. (a) 121, 6.19 (b) 14, 1703.8 (c) 1716, 3.59
(d) 1026, 58 770
13. (a) Frequency = 5 + 18 + 22 + 28 + 22 + 18 + 5 = 118
(Area = f.d. × width)
(b) Symmetric. Midpoints of intervals have been taken to represent the interval.
(c) 3.5 mm (2 s.f.)
14. 28.15, 3.84
15. 5.3
16. 30.0 mph, 5.85 mph

Exercise 1g Mean and standard deviation (page 50)

1. 19
2. 8
3. 7
4. 3.74
5. $a = 6$, $b = 4$
6. 15.6, 7.66
7. 12
8. 15, 7
9. 25.9, 1.99
10. 2.3, 1.41
11. 11.7%, 2.2%
12. (a) 4.6, 2 (b) 4.56, 2.04
13.

25	1 2 4 4
30	0 1 1 2 2 2 3 3 3 4
35	0 1 2 3 3 3 3 4
40	0 2 2 4
45	0 4
50	2
55	
60	1

Key: 45 | 4 means 49

Features: modal class 30–34, skewed to the right, 61 extreme value (outlier), 36.87, 35.59

14. £195.45, £14.12
15. 11.87, 0.80
16. 4.44

Exercise 1h Scaling sets of data (page 55)

1. (a) 6, 2.14 (b) 516, 2.14 (c) 78, 27.8
2. 50, 12
3. (a) $\mu + k, \sigma$ (b) $p\mu, p\sigma; 3\mu + 5, 3\sigma$
4. (a) 2 (b) 200 (c) 2.02 (d) −4, −1, 2, 5, 8, 11, 14
5. (a) $a = \frac{3}{4}$, $b = 22$ (b) 70 (c) 76
6. (a) 38, 8.99 (b) 34, 77
7. $a = 1.6$, $b = 10$
8. $a = 0.8$, $b = -5$; 6.25
9. (b) Take mark intervals $0 \leqslant$ mark < 10, $10 \leqslant$ mark < 20, etc.
   f.d. 0.1, 0.8, 1.9, 2.8, 2.5, 1.7, 0.7, 0.3, 0.1.
   boundaries 0, 10, 20, 30, 40, 50, 60, 70, 80
   (c) midpoints 5, 15, 25, etc, 40.4, 15.4;
   (d) $a = 24$, $b = 0.65$ (2 s.f.)
10. (a) 12.5 (b) 20; 80, 5.

Exercise 1i Coding (page 58)

1. (a) 313.76, 5.19 (b) 431, 132 (c) 0.0171, 0.00818
2. 51.235, 0.927
3. 89.3275
4. 31.7 mins.
5. 71.2, 3.82
6. $46\frac{2}{3}$ secs.

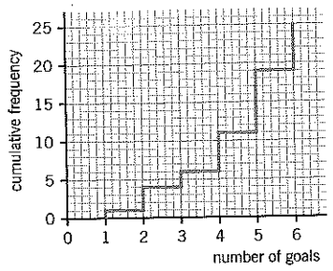
Exercise 1j Cumulative frequency, median and quartiles – ungrouped data (page 73)

1. (a) 9 (b) 207 (c) 1896 (d) 0.55
2. 4
3. (a) 61 (b) 52 (c) 73 (d) 21
4. (a) 46, 35 (b) 1.8, 1.2 (c) 20.5, 11.5
5. (a) 7, 2 (b) 14, 3

6. (a)

number of goals	0	⩽1	⩽2	⩽3	⩽4	⩽5	⩽6
cumulative frequency	0	1	4	6	11	19	25

(b)



(c) 5
(d) 2

7. (a) 2 (b) 3 (c) 2.47 (d) 1.94
8. (a) 2, 3 (b) 2
   (c) It only considers the middle 50% and does not take account of large families.

Exercise 1k Cumulative frequency, median and quartiles – grouped data (page 81)

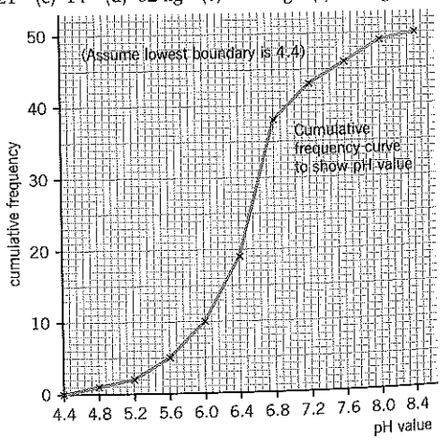
Some answers are approximate and depend on the curve drawn

1. (a)

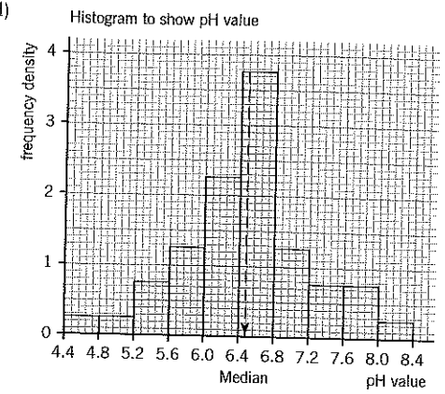
mass (kg)	cumulative frequency
⩽39.5	0
⩽44.5	3
⩽49.5	5
⩽54.5	12
⩽59.6	30
⩽64.5	48
⩽69.5	51
⩽74.5	52

Plot
(39.5, 0), (44.5, 3), (49.5, 5), (54.5, 12), (59.5, 30), (64.5, 48), (69.5, 51), (74.5, 52). Join with a smooth curve.
(b) 21 (c) 14 (d) 62 kg (e) 58.4 kg (f) 7.2 kg

2. (a)



(b) 82%
(c) 6.5, median
(d)



3.

mass (g)	cumulative frequency
⩽50	3
⩽54	5
⩽58	10
⩽62	22
⩽66	32
⩽70	38
⩽74	40

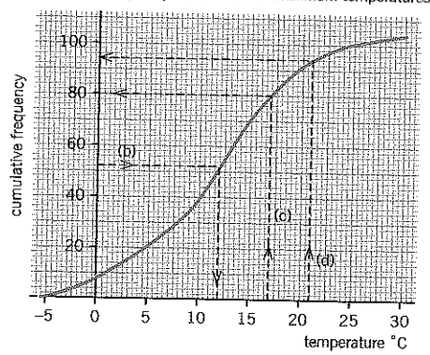
Plot (50, 3), (54, 5), (58, 10), (62, 22), (66, 32), (70, 38), (74, 40)
Median mass = 61.3 g

4. (a)

time (minutes)	cumulative frequency
⩽5	2
⩽10	4
⩽15	7
⩽20	13
⩽25	25
⩽30	41
⩽35	47
⩽40	50

(b) 24 (c) 26 (d) 23 (e) 25 mins (f) 4.5 mins

5. (a) 687.5 hours (b) 13.2 hours
6. (a) 80.75 g (b) 215
7. (a)



(b) 12°C (c) 80 (d) Approx. 10%

8.

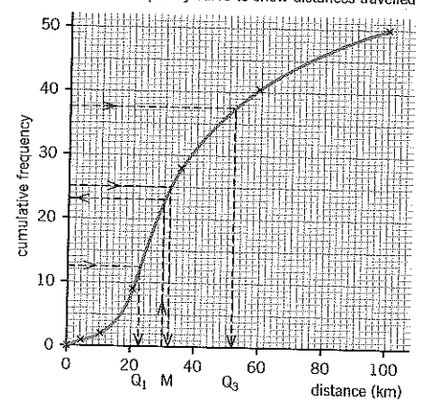
time (mins)	cumulative frequency
< 39.5	0
< 44.5	8
< 49.5	30
< 54.5	64
< 59.5	94
< 64.5	120

For the curve, plot (39.5, 0), (44.5, 8), (49.5, 30), (54.5, 64), (59.5, 94), (64.5, 120) and join points with a smooth curve.
(a) 9 mins (b) Approx. 11%; 56 mins

9.

distance (km)	cumulative frequency
0	0
< 4	1
< 10	3
< 20	9
< 35	28
< 60	40
< 100	50

Cumulative frequency curve to show distances travelled



(a) 32 km (b) Approx. 30 km (c) Approx. 54%

10.

price (£x)	cumulative frequency
⩽75	0
⩽95	6
⩽100	16
⩽105	28
⩽110	41
⩽120	48
⩽135	53

Plot (75, 0), (95, 6), (100, 16), (105, 28), (110, 41), (120, 48), (135, 53)
(a) £104 (b) £13 (c) 47

11. $x = 25$, $y = 17$
12. Plot (405, 0), (415, 4), (425, 7), (435, 13), (445, 23), (455, 28), (465, 30).
    437, 412.5, 453.
13. Plot (80, 0), (85, 6), (90, 18), (95, 40), (100, 71), (105, 86), (110, 93), (115, 97), (120, 99), (125, 100)
    (a) 97 mins (b) 10 mins (c) 62
14. Plot (165, 0), (170, 18), (175, 55), (180, 115), (185, 180), (190, 228), (195, 250)
    (a) 180.5 cm (b) 175.5 cm (c) 187 cm (d) 189.5 cm
15. (a) 57 mins (b) 71.5 mins (c) 32%.

16. Plot (69.5, 0), (74.5, 8), (79.5, 28), (84.5, 53), (89.5, 84), (94.5, 94), (99.5, 100).
9.3 secs, 22, 75.5 secs.
17. 50p, £4.96, £5.96. Large amounts affect the mean but not the median
18. Histogram: frequency densities 0.2, 0.5, 0.9, 0.8, 0.1

thickness (mm)	0	< 20	< 30	< 40	< 50	< 60
cumulative number of strata	0	2	7	16	24	25

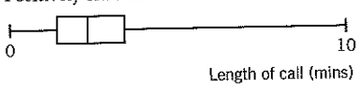
Plot (0, 0), (20, 2), (30, 7), (40, 16), (50, 24), (60, 25)
36 mm, 15 mm, 0.24.

Exercise 1l Skewness (page 90)

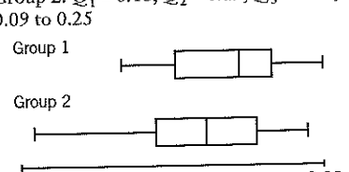
1. (a) 0.535 (b) −0.674
2. −2.4
3. 2
4. (a) Frequency densities: 0.8, 3, 5, 1.8, 1.2, 0.47, 0.2
(b) Positively skewed
5. Vertical line graph, 2, 3, 3.53, 1.985, 0.801, 0.771
6. −0.482
7. (a) *B* (b) *A* (c) *C*
8. (a) (i) 0.75 (ii) 0.28
(b) Frequency densities: 0.2, 1, 1.2, 1.8, 2.8, 0.6, 1.2, 1, 0.4, 0.2
9. (a) 9.6 mins, 1 min (b) 0.33
(c) (4.65 mins, 14.61 mins)
(d) (4.3 mins, 15.27 mins)
10. (a) 0.143 ($Q_1 = 17$, $Q_2 = 26$, $Q_3 = 38$)
(b) 0.0668 ($Q_1 = 11.9$, $Q_2 = 16.1$, $Q_3 = 20.9$)
(c) 0.333 ($Q_1 = 9$, $Q_2 = 11$, $Q_3 = 15$).

Exercise 1m Box plots (page 99)

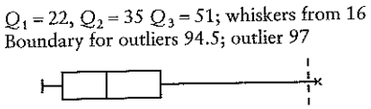
1. (a) Plot (0, 0), (1, 8), (2, 19), (3, 36), (5, 44), (10, 50)
(b) 2.35 mins, 1.4 mins, 3.4 mins
(c) Positively skewed



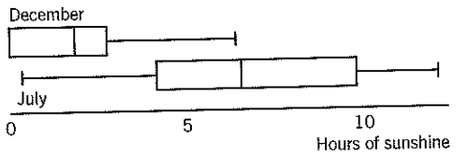
2. Group 1: $Q_1 = 0.17$; $Q_2 = 0.21$, $Q_3 = 0.23$; times from 0.14 to 0.26
Group 2: $Q_1 = 0.16$, $Q_2 = 0.19$, $Q_3 = 0.22$; times from 0.09 to 0.25



3. $Q_1 = 22$, $Q_2 = 35$ $Q_3 = 51$; whiskers from 16 to 97.
Boundary for outliers 94.5; outlier 97



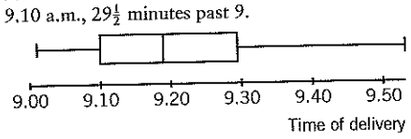
4. (a) u.c.b. 0, 20, 30, 40, 50; c.f. 0, 20, 40, 65, 69; $Q_1 = 17.5$, $Q_2 = 27.5$, $Q_3 = 35$; 7.5, 10; negatively skewed
(b) u.c.b. 0, 20, 40, 80, 100; c.f. 0, 4, 10, 34, 44; $Q_1 = 41.7$, $Q_2 = 60$, $Q_3 = 78.3$; 18.3, 18.3; negatively skewed, zero quartile skewness
(c) u.c.b. 0, 5, 10, 15, 20, 25, 35; c.f. 0, 1, 6, 9, 11, 12, 13; $Q_1 = 7.25$, $Q_2 = 10.8$, $Q_3 = 16.875$; 6.075, 3.55; positively skewed
(d) u.c.b. 0, 5, 10, 15, 20, 25, 30; c.f. 0, 5, 20, 45, 90, 140, 160; $Q_1 = 14$, $Q_2 = 18.9$, $Q_3 = 23$; 4.1, 4.9; negatively skewed
5. $\bar{x} = 63.9$, $s = 29.5$, outliers would be less than 4.9 mins, greater than 122.8 mins, outliers are 133, 144.
6. Compare median, quartiles, range, skewness
7. December: $Q_1 = 0.3$, $Q_2 = 1.8$, $Q_3 = 2.7$;
July: $Q_1 = 4.1$, $Q_2 = 6.5$, $Q_3 = 9.8$



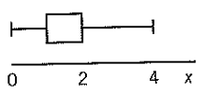
8. (a)

0	1 2 2 5 9
1	0 0 2 3 5 7 9 9
2	2 5 9 9 9
3	0 1
4	5 7 8
5	3

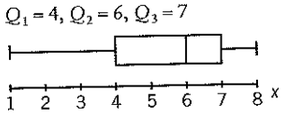
Key: 2 | 5 means 9.25 a.m.
(b) 9.19 a.m.
(c) 9.10 a.m., 29½ minutes past 9.
(d)



9. (a) $Q_1 = 1$, $Q_2 = 1$, $Q_3 = 2$



(b) $Q_1 = 4$, $Q_2 = 6$, $Q_3 = 7$



10. (a) 6, 5
(b) More than 3 standard deviations from the mean
(c) (i) older brother or sister also attended
(ii) a mistake had been made
(d) 5.5, 5
(e) decrease
(f) positive, less
11. (a)

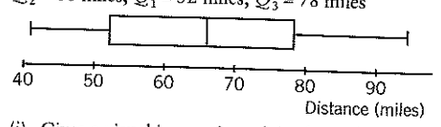
	height gain (grams)
36	0 9 9
37	6
38	
39	1 7 7 9
40	2 3 7
41	0 0
42	0 5 7
43	0 4
44	5
45	
46	2

Key: 39 | 7 means 397
(b) Draw plots – New corn: whiskers from 360 to 462, $Q_1 = 397$, $Q_2 = 450$, $Q_3 = 426$; Standard corn: whiskers from 321 to 423; $Q_1 = 353$, $Q_2 = 368.5$, $Q_3 = 383$

12. (a)

Stem	Leaf
4	1 2 3 4 4 6 7 7 8 8
5	0 2 2 2 3 4 6 7 8 8
6	0 2 3 3 6 6 7 7 8
7	0 0 2 2 4 4 6 7 8 8 8
8	0 1 2 5 5 6 6 7
9	3 3 4

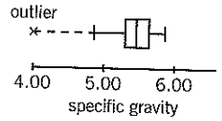
Key: 4 | 2 means 42
(b) $Q_2 = 66$ miles, $Q_1 = 52$ miles, $Q_3 = 78$ miles
(c)



(d) (i) Gives a visual impression of the data whilst keeping the details.
(ii) Gives an immediate impression of an approximately symmetrical distribution with the middle 50% lying between 52 and 78 miles.

Miscellaneous exercise 1n (page 110)

1. (a) $\bar{x} = 5.42$, $s = 0.33$; range = 1.79, $Q_2 = 5.46$, $Q_1 = 5.295$, $Q_3 = 5.615$, outlier = 4.07
(b) (i) 5.465
(ii) 5.47
(iii) 0.22

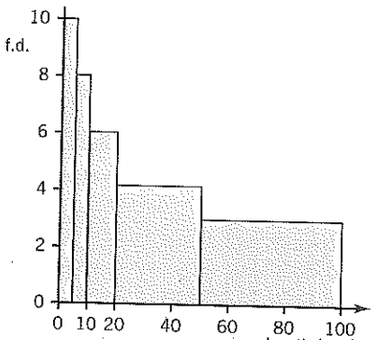


2. (a) Boundary points for histogram
689.5, 709.5, 719.5, 729.5, 739.5, 744.5, 749.5, 754.5, 759.5, 769.5, 789.5
First interval l.c.b. 689.5, u.c.b. 709.5
f.d. 0.15, 0.7, 1.5, 3.8, 8.2, 7, 4.2, 3.2, 1.4, 0.5
(b) Plot (689.5, 0), (709.5, 3), (719.5, 10), (729.5, 25), (739.5, 63), (744.5, 104), (749.5, 139), (754.5, 160), (759.5, 176), (769.5, 190), (789.5, 200).
(c) 744.24, 14.86
(d) 744.01, 736.08, 752.12
(e) 0.046
(f) 0.011
(g) In box plot, draw whiskers from 689.5 to 789.5, with median and quartiles as in (d).
3. 16, 6 (a) 5.86 (b) 15, 7
4. 35 yrs 1 month, 11 yrs 3 months.
(a) median = 33 yrs 9 months, IQR = 17 yrs 11 months
(b) 61.8%
5. (a) 44.5
(b) 51.75
(c)

$Q_1 = 40$, $Q_3 = 64$
(d) $a = 0.85$, $b = 1$ (dependent on values in (c))
(e) yes
6. (a) (i) 49.66 (ii) 433.97 (iii) 20.83
(b) c.f. 3, 9, 18, 28, 40, 58, 72, 83, 88
(c) Plot (0, 0), (10, 3), (20, 9), (30, 18), (40, 28), (50, 40), (60, 58), (70, 72), (80, 83), (90, 88)
(d) (i) 52 (ii) 32
(e) 11
7. c.f. 11, 39, 77, 111, 138, 150
Take as boundaries 0.90, 1.15, 1.30 etc. or 0.91, 1.16, 1.31, etc. or 0.905, 1.155, etc. Median ≈ £1.30.
8. (a)

Stem (£)	Leaf (p)
3	40 60 75 95
4	20 50 75
5	20 75
6	45 60
7	25
8	75
9	60
10	
11	
12	25

$Q_2 = £5.20$, range = £8.85
(b) $\bar{x} = £6$, $s = £2.47$
(c) A: $\bar{x} = £6.30$, $s = £2.47$
B: $\bar{x} = £6.30$, $s = £2.59$
(d) mean remains the same; lower paid workers do not benefit under scheme B.
9. (a) 8, 6, 4⅙, 3



(c) Approx. 2.5 mm (modal class is $0 \leqslant x < 5$)
(d) (i) 39.9 mm
(ii) 35 mm
10. (a) 275
(c) Comparative bar chart
11. 57, (a) it becomes 39
(b) $x = 3x - 141$ does not have an integer solution.
12. 100.7 mm, 0.4 mm; machine B nearer 100 on average, less variation with machine A.

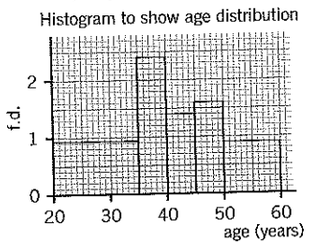
13. (a) 1, could be 1 or 2
(b) Positive skew, possible outlier
(c) 2, 1.7; more than 3 standard deviations from the mean
(d) (A) a mistake.
(B) could be correct.
(e) 1.88, 1.48

14.

Cost (£1000)	⩽ 50	⩽ 60	⩽ 70	⩽ 100	⩽ 150
c.f.	540	1690	3010	3870	4320

Plot (20 000, 0), (50 000, 540), (60 000, 1690), (70 000, 3010), (100 000, 3870), (150 000, 4320).
$Q_2 \approx$ £63 000, IQR: a value between £18 000 and £23 000 is acceptable

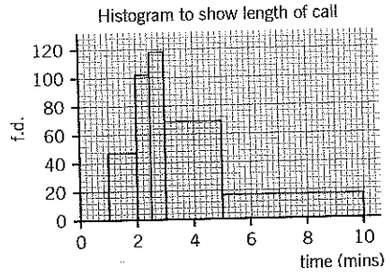
15. f.d. 0.93, 2.4, 1.4, 1.6, 0.9



(a) 40.15 (b) $35\frac{1}{2}$ yrs.

16.

Time (mins)	Frequency	Frequency density
$0 \leqslant x \leqslant 1$	20	20
$1 < x \leqslant 2$	47	47
$2 < x < 2.5$	51	102
$2.5 < x < 3$	59	118
$3 < x \leqslant 5$	138	69
$5 < x \leqslant 10$	85	17



$3\frac{1}{3}$ mins, divides area in half.

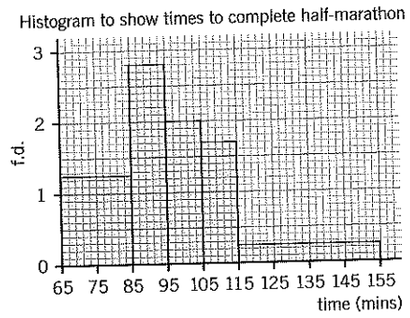
17. (a) 8, 9.5 mins
(b) Boundaries 0, 5, 10, 15, 20, 25, 30; f.d. 8, 11.2, 5.6, 4, 2.4, 0.8
(c) 10
(d) A False, B True, C False, D True

Mixed test 1A (page 114)

1.

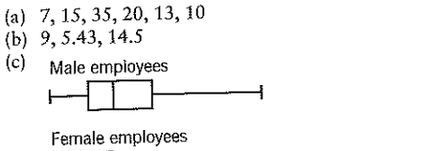
t	f	f.d.
$65 < t \leqslant 85$	25	1.25
$85 < t \leqslant 95$	28	2.8
$95 < t \leqslant 105$	20	2
$105 < t \leqslant 115$	17	1.7
$115 < t \leqslant 155$	10	0.25

(a)



(b) 96.15 mins

2. (a) 7, 6, 4, 8
(b) 6.55, 5.7, 8.1
(c)



(d) Positive skew.

3. (a) 4.5 (b) 1.5
(c) No change to mean, standard deviation is increased.

4. (a) Pie chart, bar chart
(b) Children in school, sample not representative.
(c) f.d. 3.6, 6.4, 4.4, 1.4, 0.7

NB: boundary points could be 0.5, 5.5, 10.5, 15.5, 25.5, 45.5
(d) 13.8, 10.2
(e) 9.11

5. (a) Histogram
(b) Individual values are not known and mid points have been taken as representatives of the intervals.
(c) 69.5, 7.6
(d) Median – no effect, IQR – no effect, mean – increased.

6. (a) 7, 15, 35, 20, 13, 10
(b) 9, 5.43, 14.5
(c)

Mixed test 1B (page 116)

1. (a) 1.15 (b) 1 (c) 1.09

2. (a) (i) Easier to see the spread
(ii)

1	1 2 2 3 4 4 4
1	5 6 7 7 9
2	1 1 1 2
2	5 5 7
3	1 2 2
3	5 5 9
4	1 3
4	4 5

Key: 1 | 5 is 15 cm

(b) 24.6 cm
(c) 21 cm
(d) Median better; distribution not symmetrical.

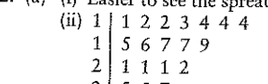
3. (a) 44%
(b) 33°

4. (a) Median same for both.
B has 3 outliers; ignoring these, B's average waiting time would be lower.
B's times are less variable than A's.
A's times are positively skewed, B's are negatively skewed.
(b) (i) If outliers are not the Post Office's fault, choose B for quicker service.
(ii) If outliers are the Post Office's fault then the situation could happen again and there could be a long wait. A avoids long waits.

5. (a)

00	6 7 8 8
10	0 0 1 2 2 3 3 3 4 4
10	5 6 6 6 6 7 7 7 8 8 9 9
20	0 2 3
20	7

(b) $Q_2 = 15.5$ mins, $Q_1 = 12$ mins, $Q_3 = 18$ mins.
(c)

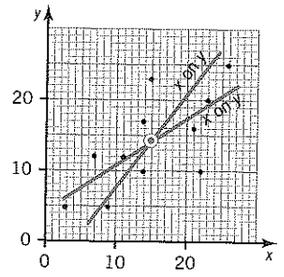


6. (a) (i) 3 hrs 3 mins
(ii) $Q_1 = 2$ hrs 42 mins, $Q_3 = 3$ hrs 42 mins.
(b) 40, (200), 200, 60
(c) (i) 3 hrs 20 mins (ii) 54 mins

Chapter 2

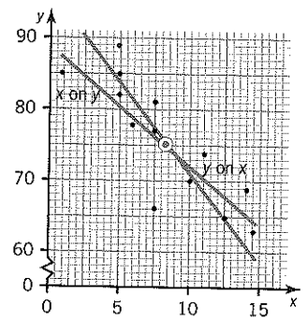
Exercise 2a Equations of least squares regression lines (page 136)

1. Data set 1
(a) $y = 4.50 + 0.64x$
(b) $x = 4.42 + 0.75y$



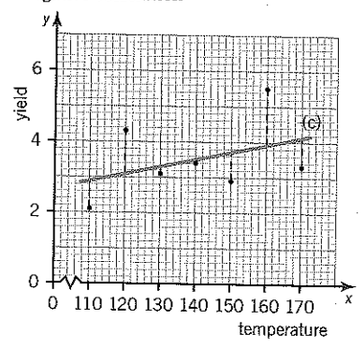
Good positive correlation

Data set 2
(a) $y = 90.31 - 1.78x$
(b) $x = 37.80 - 0.39y$



Good negative correlation

2. (a)



(b) $y = 0.614 + 0.0207x$

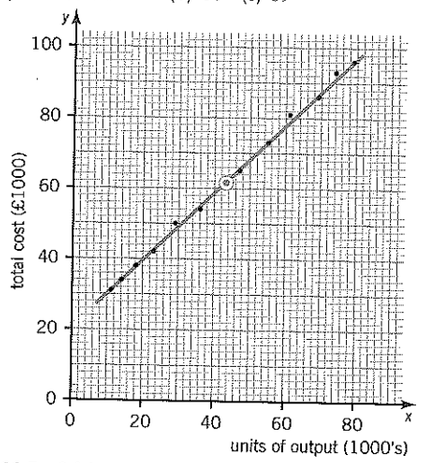
3. $y = -2.59 + 0.65x$; 36.5

4. $F = -6.33 + 0.90I$, $F = 20.8$

5. $y = 3.8 + 1.6x$, $x = -2.06 + 0.59y$

6. (a) $y = 15.83 + 0.72x$ (b) 66 (c) 59

7. (a)



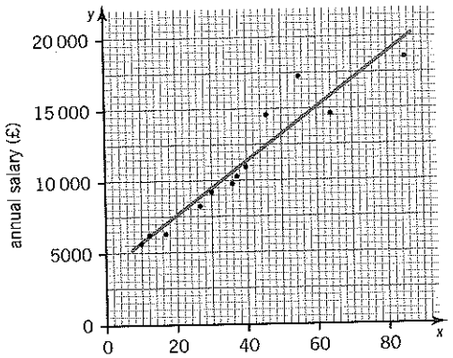
(b) $20.7 + 0.96x$
(c) 31 000 – 33 000 units (d) Break-even point.

8. $y = 1.8 + 1.3x$

9. $y = -8 + 1.2x$

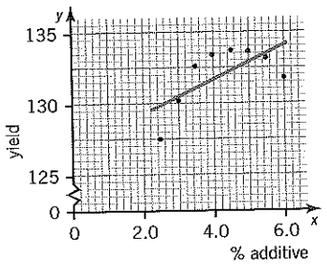
10. $c = 15$, $d = -5$

11. (a)

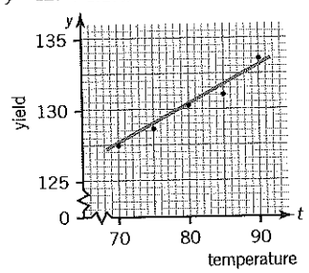


(b) $y = 3710 + 192x$
(c) Appears reasonably satisfactory apart from B and C who have earned substantially more than the equation suggests.
(d) (i) $y = 4210 + 192x$
(ii) $y = 4010 + 207x$
(iii) $y = 4160 + 200x$
(e) It would contain a term for employees who work away from home e.g. $y = a + bx + c$, where $c \approx £3000$ for employees who work away from home and zero otherwise.

12. 0.3, 0.6
13. (a)



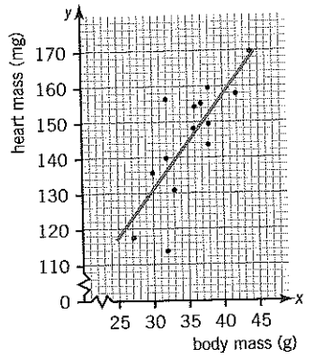
(b) $y = 127 + 1.17x$
(c)



(d) Argument invalid since relationship between yield and additive is not linear, yield declines above 4.5% additive; suggest additive 4.5%, temperature 90°.

Exercise 2b Product-moment correlation coefficient (page 145)

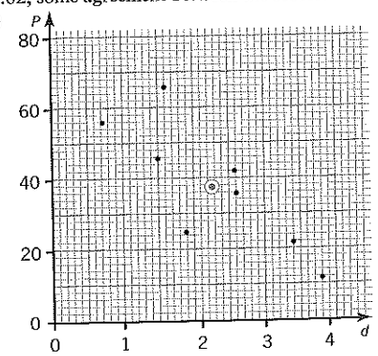
1. (a) 0.930, strong positive correlation
(b) −0.828, strong negative correlation
(c) 0.867, strong positive correlation
(d) 0.742, positive correlation.
2. 0.82
3. (a) −0.558
(b) Low unemployment appears to be linked to high wage inflation, so suggestion justified.
4. 0.79
5. 0.73, $y = -25.4 + 0.53x$, $x = 94.4 + 1.01y$
6. 0.60, $W = -76 + 0.89\,h$
7. 0.77
8. −0.415
9. (a) 0.954 (b) 2, 3
10. (a)



(b) $y = 48.35 + 2.75x$
(c) 0.787

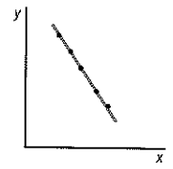
Exercise 2c Spearman's coefficient of rank correlation (page 151)

1. 0.26
2. (a) 0.43
(b) Some agreement between average attendance ranking and position in league, high position in league correlating with high attendance.
4. 0.033, little or no correlation.
5. −0.62, some agreement between the scores.
6. (a)

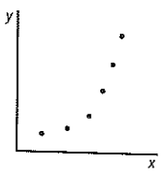


(b) (2.275, 38.375)
(c) Ranking both p and d from lowest to highest gives −0.839.
(d) In general the population density is greater nearer the centre of the town and less on the outskirts of the town.
(e) H, low population density and distance from centre of town.
7. (a) 0.3, 0.5, 0.7
(b) Mrs Brown and John; 1) Headrests 2) Heated rear window 3) Anti-rust treatment.

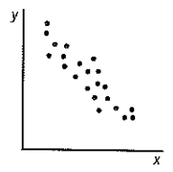
8. −0.036, no agreement.
9. 0.84, strong positive correlation between number of years smoking and extent of lung damage.
10. (a)

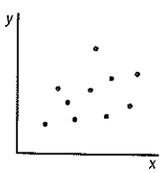


(b)

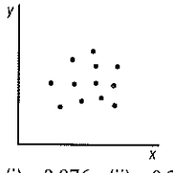


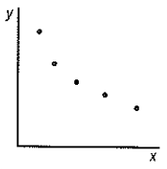
(c) −0.92
(d) −0.9
11. (a)





0.60, 0.60
12. (a) 0.7, good agreement between judges.
(b)

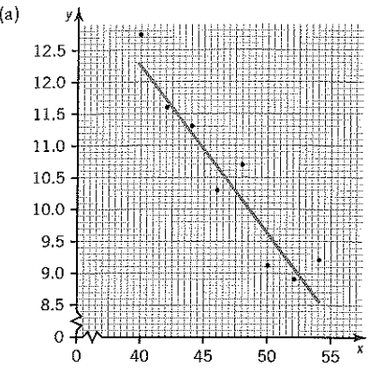




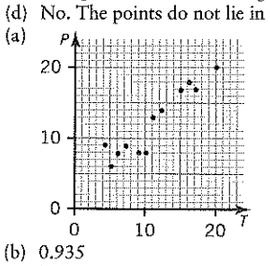
13. (a) (i) −0.976 (ii) −0.292 (or 0.292)
(b) The transport manager's order is more profitable for the seller, saleswomen is unlikely to try to dissuade.
(c) (i) No, maximum value is 1
(ii) Yes, higher performing cars generally do less mileage to the gallon.
(iii) No, the higher the engine capacity, the dearer the car.
(d) When only rankings are known; when relationship is non-linear.
14. 0.84; very good agreement between the rankings indicating strong positive correlation between the marks in English and the marks in History; E.

Miscellaneous exercise 2d (page 160)

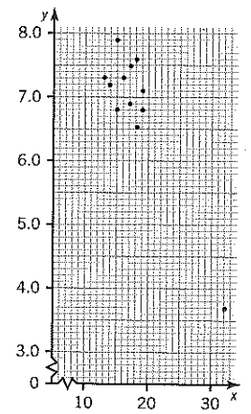
1. (a) $y = 3.07 + 1.17x$
(b) When the y variable is the controlled or independent variable.
2. (a) t on w is required; $t = 18.8 - 0.853w$
(b) (i) −13.6° F (ii) −28.1° F
(c) −0.946, points lie close to the regression line.
(d) Good estimate for $w = 38$, since strong correlation. Estimate for $w = 55$ needs to be treated with care since extrapolation (outside range of data) is unreliable.
3. (a) Strong negative correlation
(b) $y = 6.85 - 0.0072x$
(c) pH = 6.85 at $t = 0$°C; for an increase of 10°C, pH drops by 0.07
(d) 6.71, reliable; 6.17, unreliable, outside range of data
(e) 48.6°C
4. (a)



(b) $y = 23.0 - 0.267x$
(c) 7500. There isn't a wide degree of scatter, so estimate could be reliable, but in general it is unwise to extrapolate outside the range of data.
(d) No. The points do not lie in a line.
5. (a)

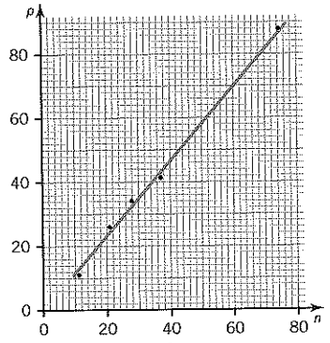


(b) 0.935
(c) b) indicates strong positive linear correlation and diagram confirms this is appropriate.
(d) $p = 2.58 + 0.88T$; 15
6. (a) Σm^2, page 121 diagram 3
(b) $y = 7.77 - 0.005x$
(c) 5.77; treat with caution as outside range of data.
(d) The lower the percentage moisture content, the greater the heat output.
7. (a) −0.901, strong negative correlation, the greater the number of items finished, the lower the mean quality score.

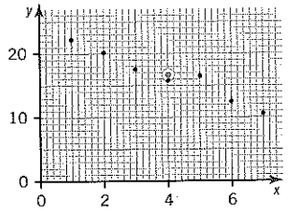


(b) Amend; possibly negative trend but not strong correlation, (32, 3.7) is an outlier
(c) Ignore outlier; weak negative correlation between number of items and quality score.

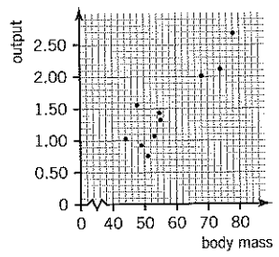
8. $y = 0.65 + 0.0157x$;
Rate of about 1 hour per mile distance; 3 days 19 hours; out of range of data, travel across water required; 0.942, strong positive correlation, points close to regression line.
9. (a) $y = 12.033 - 0.009x$
(b) 8.6 per 1000
(c) Decreasing number of members of population per doctor not effective in reducing infant mortality rate.
10. (a) Spearman 0.613; grades given
(b) Product-moment, 0.95; numerical data given
(c) Students performed at a similar standard in the written and listening tests, but not in the oral test. Standard in oral test related more to listening performance than written performance.
11. (a)



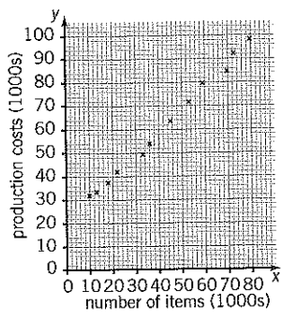
(b) $p = -0.54 + 1.2$ n; £17
(c) 0.998; the points will be close to a line with positive gradient.
12. (a) 0.96
(b) points lie close to a straight line with positive gradient (strong positive correlation).
(c) Equal to 0.986 since rankings will not change.
13. (a) 0.0705 (b) $y = 0.34 + 0.0085x$
(c) 0.477 (d) unreliable since outside range of data
14. (a)



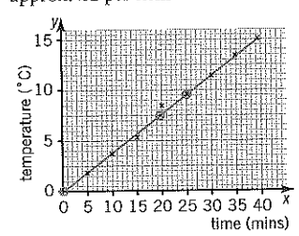
Mean (4, 16.7).
(b) average decrease of 1.80°C per month
(c) $y = 23.9 - 1.80x$
(d) 23.9°C; regression line is valid only within range of data.
15.



0.91, strong positive correlation.
16. (a) 0.98 (b) $y = -7.42 + 1.115x$, $x = 6.82 + 0.862y$
(c) 8.20 tons per acre (d) 13.9 cm
17. (a) $y = 41.79 + 1.55x$ (b) 51
(c) 43, but treat with caution as outside range of data.
18. (a) −0.9 (b) 0 (c) 0.9 (0.6 without outlier)
19. (a)



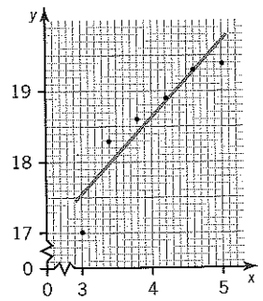
(b) Diagram suggests a linear relationship
(c) $y = 61.1 = 0.966(x - 42.4)$
(d) $y = 20.1 + 0.966x$
(e) Initial costs are approx. £20 000, cost increases by approx. £1 per item
20. (a)



(b) $y = -0.142 + 0.389x$
(c) 23.2°C, outside range

Mixed test 2A (page 166)

1. (a) $y = 3.667 + 0.038x$ (3 d.p.)
(b) Mathematically $a = 3.667$ would indicate a yield of 3.667 tonnes with no water at all; in practice this would be nonsense, $b = 0.038$ indicates yield increases by 0.038 tonnes for every extra centimeter of water.
(c) 4.7 tonnes (only just outside range, probably reliable), 9.3 tonnes (well outside range, unreliable).
2. (a) $y = 14.55 + 1.02$ t
(b) Initial temperature of milk.
(c) 19.14°C, 34.95°C
(d) First; second outside range of data.
(e)

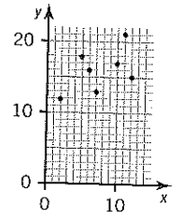


(f) Temperature would stabilise at room temperature.
(g) Points appear to lie on a curve, reaching a limit at room temperature.

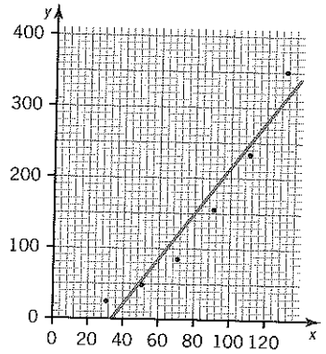
3. 0.714
4. (a) −0.975; points lie close to a straight line with negative gradient (strong negative correlation).
(b) −1, complete disagreement in the rankings.
(c) (iii), data follow a non-linear relation.

Mixed test 2B (page 167)

1. (a) −0.987, points lie close to a line with negative gradient.
(b) y on x, $y = 7.22 - 0.69x$; 4.45
(c) Depreciation of £700 per year.
(d) (i) No, outside range of data.
(ii) Yes, since x is controlled. Use $x = \frac{y-a}{b}$.
2.



0.515; 8.8 hours; regression line gives average value, points not that close to line as $r = 0.515$; Σm_i^2 minimised where m_i is vertical distance from point to line.
3. (a)



(b) $y = -107 + 3.21x$
(c) 214, overestimate (data fit a curve); 375, underestimate (outside range, also non-linear relationship), unreliable
(d) no, better to use a curve.
4. (a) 0.714
(b) Same, since there is no change in the rankings.
(c) d^2 would decrease, therefore $1 - \frac{6\Sigma d^2}{n(n^2-1)}$ would increase.

Chapter 3

Exercise 3a Elementary probability (page 173)

1. (a) $\frac{1}{3}$ (b) 1 (c) $\frac{2}{3}$
2. (a) 0.375 (b) 0.625 (c) 0.75 (d) 0 (e) 0.8
3. (a) 0.3 (b) 0.75
4. (a) 0.4625 (b) 800
5. 0.73
6. (a) (i) $\frac{1}{52}$ (ii) $\frac{7}{26}$ (iii) $\frac{10}{13}$ (b) $\frac{13}{17}$
7. $\frac{4}{15}$
8. (a) $\frac{2}{7}$ (b) $\frac{3}{7}$
9. $\frac{4}{15}$
10. 0.27 (2 d.p.)
11. (a) (i) $\frac{75}{98}$ (ii) $\frac{15}{98}$ (b) $\frac{1}{5}$
12. 0.52
13. (a) $\frac{1}{18}$ (b) $\frac{1}{6}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$
14. (a) (i) $\frac{1}{36}$ (ii) $\frac{1}{12}$ (iii) 0 (b) $t = 6$ or 12

Exercise 3b (Probability) – combined events (page 181)

1. (a) $\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{5}{6}$
2. $\frac{11}{30}$
3. (a) $\frac{4}{17}$ (b) $\frac{4}{51}$ (c) $\frac{5}{17}$ (d) $\frac{5}{17}$
4. $\frac{3}{4}$
5. (a) 0.5 (b) 0.4 (c) 0.2 (d) 0.1
6. (a) (i) $\frac{1}{3}$ (ii) $\frac{1}{9}$ (iii) $\frac{38}{45}$ (b) 0.2
7. (a) $\frac{3}{4}$ (b) 0 (c) $\frac{1}{4}$
8. 0.6
9. 0.7
10. (a) $\frac{7}{36}$ (b) $\frac{1}{6}$ (c) $\frac{5}{18}$ (d) $\frac{1}{12}$
11. $\frac{3}{4}$
12. (a) $\frac{11}{36}$ (b) $\frac{11}{36}$ (c) $\frac{5}{9}$
13. Yes
15. At least one tail is obtained; both coins show tails.
16. (a)

	Fruit tree	Other tree	Total
Birds nest	2	4	6
No nest	5	9	14
Total	7	13	20

(b) 0.45 (c) $\frac{2}{7}$

Exercise 3c Combined events (page 192)

1. (a) $\frac{1}{3}$ (b) 0
2. (a) 0.05 (b) 0.5
3. (a) 0.15 (b) 0.65; no
4. (a) $\frac{3}{10}$ (b) $\frac{1}{10}$
5. $\frac{1}{9}$
6. (a) $\frac{1}{4}$ (b) $\frac{1}{6}$
7. $\frac{1}{2}$
8. (a) 0.5 (b) 0.35 (c) 0.375 (d) 0.4
9. (a) $\frac{1}{2704}$ (b) $\frac{1}{16}$ (c) $\frac{1}{2}$ (d) $\frac{25}{169}$
10. (a)

	B	G	Totals
Passed	16	8	24
Taken, failed	7	6	13
Learning	10	8	18
Too young	2	3	5
Totals	35	25	60

(b) $\frac{13}{60}$ (c) $\frac{14}{25}$ (d) $\frac{12}{35}$ (e) $\frac{1}{177}$ (f) $\frac{128}{875}$
11. (a) Independent; obtaining a head when a coin is tossed.
(b) Mutually exclusive, 0.
12. $\frac{3}{8}$
13. 0.5
14. (a) $\frac{1}{3}$ (b) $\frac{2}{15}$ (c) $\frac{8}{15}$
15. (a) 0.2 (b) 0.03 (c) 0.32
16. (a) $\frac{1}{15}$ (b) $\frac{11}{15}$ (c) $\frac{1}{5}$
17. (a) (i) $\frac{1}{16}$ (ii) $\frac{24}{169}$ (iii) $\frac{1}{52}$ (b) (i) $\frac{32}{221}$ (ii) 0
18. (a) 0.5 (b) (i) 5p (ii) 4p (c) $\frac{1}{40}$
19. (a) 0.1 (b) 0.3 (c) 0.45
20. (a) $\frac{3}{7}$ (b) $\frac{3}{8}$
21. (a) $\frac{5}{21}$ (b) $\frac{2}{3}$ (c) $\frac{5}{12}$
22. (a) 0.15 (b) $\frac{7}{15}$ (c) $\frac{1}{75}$

Exercise 3d Tree diagrams (page 200)

Section A

1. (a) 0.0025 (b) 0.095
2. (a) $\frac{5}{14}$ (b) $\frac{17}{42}$
3. (a) 0.24 (b) 0.42
4. (a) (i) $\frac{8}{27}$ (ii) $\frac{4}{9}$ (iii) $\frac{7}{27}$
   (b) (i) $\frac{5}{21}$ (ii) $\frac{15}{28}$ (iii) $\frac{19}{84}$
5. 0.00599, 0.987
6. (a) $\frac{12}{49}$ (b) $\frac{20}{49}$
7. $\frac{7}{16}$
8. $\frac{25}{72}$
9. $\frac{5}{16}$
10. 0.35
11. 0.825
12. (a) 0.5 (b) 0.5 (c) 0.375
13. 0.788
14. (a) 0.02 (b) 0.64
15. (a) $\frac{3}{11}$ (b) $\frac{12}{55}$ (c) $\frac{3}{44}$
16. (a) 0.34 (b) 0.063 (c) 0.19 (d) 0.97; 3 white
17. 0.624
18. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{16}$ (d) $\frac{1}{4}$ (e) $\frac{3}{4}$

Section B

1. (a) $\frac{5}{12}$ (b) $\frac{3}{5}$
2. (a) $\frac{21}{38}$ (b) $\frac{15}{38}$ (c) $\frac{20}{83}$
3. (a) P(A occurs, given that B occurs)
   (i) mutually exclusive (ii) independent
   (b) 0.88, 0.05
4. (a) 0.33 (b) $\frac{7}{11}$
5. (a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{8}{9}$
6. (b) $\frac{77}{95}$
7. (a) $\frac{7}{18}$ (b) (i) $\frac{5}{8}$ (ii) $\frac{8}{25}$
8. (a) $\frac{1}{25}$ (b) $\frac{106}{125}$ (c) $\frac{14}{19}$
9. (a) 0.096 (b) 0.156; $\frac{5}{13}$
10. (a) 0.7, 0.68 (b) 0.28 (c) 0.65625
11. (a) $\frac{6}{323}$ (b) $\frac{135}{323}$ (c) $\frac{1}{5}$ (d) $\frac{1}{5}$
    (i) Yes, no (ii) No, yes
12. (a) 0.000877 (b) 0.421 (c) 0.65 (d) 0.642
13. (a) 0.042875 (b) 0.142 (c) 0.1215
    (d) 0.189 (e) 0.334125; 0.642
14. (a) (i) $\frac{9}{22}$ (ii) $\frac{6}{11}$ (iii) $\frac{2}{11}$ (iv) $\frac{4}{7}$
    (b) (i) 0.0303 (ii) 0.450 (iii) 0.0348
    (c) (i) 0.36 (ii) 0.848
15. $\frac{23}{45}$, $\frac{18}{23}$
16. (b) $\frac{9}{32}$ (c) $\frac{83}{128}$ (d) $\frac{17}{37}$
17. (a) 0.36 (b) 0.6875

Exercise 3e Useful methods (page 206)

1. (a) 0.763 (b) 14
2. (a) 5 (b) 6
3. 0.5, 6
4. 0.999
5. 22
6. $\frac{5}{11}$
7. 1 : 8
8. 0.5 (a) $\frac{1}{6}$ (b) $\frac{25}{216}$ (c) $\frac{625}{7776}$; $\frac{6}{11}$
9. (a) (i) $\frac{1}{6}$ (ii) $\frac{1}{12}$ (iii) $\frac{2}{3}$ (b) $\frac{7}{12}$

Exercise 3f Arrangements, permutations, combinations (page 219)

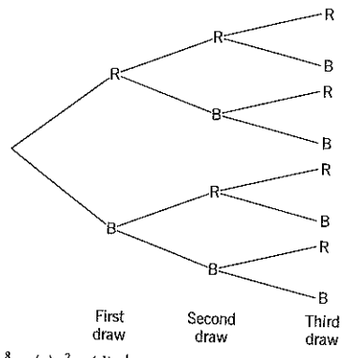
1. 9!, $\frac{1}{72}$
2. (a) 6! (b) $\frac{1}{3}$
3. (a) 4! 9! (b) $\frac{54}{55}$
4. $\frac{9}{11}$
5. $\frac{1}{126}$
6. (a) 8! (b) $\frac{1}{28}$
7. (a) $\frac{12!}{(2!)^4}$ (b) $\frac{1}{66}$
8. $\frac{28}{153}$
9. $\frac{49}{143}$
10. $\frac{60}{143}$
11. (a) 210 (b) $\frac{2}{13}$ (c) $\frac{1}{30}$
12. 12
13. (a) $\frac{1}{14}$ (b) $\frac{3}{7}$ (c) $\frac{1}{30}$
14. (a) 65 268 (b) 4263
15. 510
16. $\frac{37}{42}$
17. 4608
18. (a) 1260 (b) 2520
19. (a) 420 (b) B 252, G 462 (c) 120 (d) $\frac{44}{133}$
20. (a) 5040 (b) 1680 (c) 672
21. 5005, 720, 72
22. 5040 (a) 144 (b) 1440
23. (a) 2.5×10^{-7} (b) 3 193 344
24. (a) $\frac{1}{5}$ (b) $\frac{1}{5}$
25. 130
26. (a) 360 (b) 6 (c) 12 (d) 1170
27. (a) 64 (b) 18 (c) $\frac{21}{32}$
28. (a) 9! (b) $\frac{7}{36}$ (c) 1260 (d) $\frac{5}{9}$
29. (a) 75 (c) $\frac{181}{456}$ (d) (i) 6! (ii) 72
30. 70, (a) 55
    (b) 30
    (c) 65
    (d) $\frac{2}{7}$
    (e) $\frac{1}{7}$
    (f) $\frac{1}{7}$

Miscellaneous exercise 3g (page 228)

1. (a) 0.36 (b) 0.48 (c) 0.01024 (d) 0.98976
2. (a) C, C′ (b) C, D (c) C, E
3. (a) 0.0902 (b) unsatisfactory test
4. 0.32, 0.467
5. (a) 0.325 (b) $\frac{51}{260}$ (c) $\frac{5}{13}$
6. (a) 0.28 (b) (i) 0.157 (ii) 0.363 (iii) 0.163
   (c) 0.0728 (d) 0.404
7. 0.166, 0.580
8. 5040 (a) 720 (b) 1440
9. (a) $\frac{1}{343}$ (b) $\frac{1}{49}$ (c) $\frac{30}{49}$ (d) $\frac{8}{343}$ (e) $\frac{1}{4}$ (f) 6
10. (a) $\frac{11}{24}$ (b) $\frac{11}{60}$ (c) $\frac{43}{120}$ (d) $\frac{49}{144}$
11. (a) (i) 0.005 (ii) 0.0955 (b) 0.999 (c) 0.136
12. (a) (i) $\frac{1}{3}$ (ii) $\frac{7}{9}$ (iii) $\frac{1}{3}$ (b) $\frac{2}{15}$ (c) $\frac{3}{10}$
13. 5005, 1960, 315 (a) $\frac{9}{56}$ (b) $\frac{27}{56}$
14. (a) 792 (b) 210 (c) $\frac{35}{132}$ (d) 120 (e) 0.1 (f) 0.1
15. (a) 40 320 (b) (i) 1440 (ii) 5760
    (c) (i) $\frac{1}{7}$ (ii) $\frac{6}{7}$ (d) 576 (e) $\frac{3}{35}$
16. (a) $\frac{1}{4}$ (b) $\frac{5}{14}$ (c) independent
    (d) $\frac{1}{7}$, $P(A|C) \neq P(A)$ (e) $\frac{7}{18}$

Mixed test 3A (page 231)

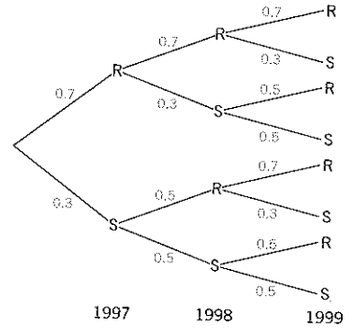
1. (a) $\frac{1}{28}$ (b) $\frac{1}{13}$
2. (a)



   (b) $\frac{8}{13}$ (c) $\frac{2}{5}$ (d) $\frac{1}{2}$
3. (a) 0.4 (b) 0.2 (c) $\frac{13}{30}$
4. (a) $\frac{23}{30}$ (b) $\frac{119}{150}$ (c) $\frac{31}{150}$ (d) 30
   (e) The probability that a female employee is weekly paid. (f) 0.5
5. (a) $\frac{1}{16}$ (b) $\frac{7}{16}$ (c) $\frac{3}{32}$ (d) $\frac{29}{32}$

Mixed test 3B (page 232)

1. (a) 0.64 (b) 0.75
2. (a) $q - 0.25$ (b) $\frac{2}{3(4q-1)}$ (c) $\frac{13}{20}$
3. (a) 0.857 (b) 0.135 (c) 0.13917 (d) 0.973
4. (a) 0.1792 (b) 0.1686 (c) 0.203
5. (a)

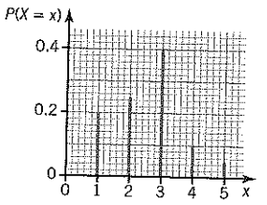


   (b) 0.372
   (c) $\frac{49}{124}$
   (d) 8

Chapter 4

Exercise 4a Probability distributions (page 236)

1 (a) 0.1



   (b) (i) 0.85 (ii) 0.55 (iii) 0.5 (iv) 3
2.

x	12	13	14
$P(X=x)$	$12k$	$13k$	$14k$

, $k = \frac{1}{39}$

3. 0.1
4. (a) $\frac{7}{20}$ (b) $\frac{1}{4}$ (c) 0 (d) $\frac{13}{20}$
5. (a)

x	0	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

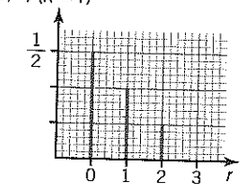
   (b)

x	0	1	2	3
$P(X=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

6.

x	0	1	2	3
$P(X=x)$	$\frac{1}{27}$	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{27}$

7. (a) $\frac{1}{6}$ (b) $P(R = r)$



   (c) $\frac{1}{2}$
8. $P(X = x) = 0.1, x = 0, 1, 2, \ldots, 9$
9. (a)

x	0	1	2	3
$P(X=x)$	0.216	0.432	0.288	0.064

   (b) 0.648
10.

x	−5	5	15
$P(X=x)$	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

11.

x	1	2	3	4	5	6
$P(X=x)$	$\frac{6}{72}$	$\frac{7}{72}$	$\frac{8}{72}$	$\frac{9}{72}$	$\frac{10}{72}$	$\frac{11}{72}$

x	7	8	9	10	11	12
$P(X=x)$	$\frac{6}{72}$	$\frac{5}{72}$	$\frac{4}{72}$	$\frac{3}{72}$	$\frac{2}{72}$	$\frac{12}{72}$

   $\frac{11}{18}$; Equally likely outcomes
12. For $x = 8$, draw vertical line to 0.2; for $x = 9$, draw vertical line to 0.3; symmetrical distribution.

Exercise 4b Expectation (page 244)

1. 2.25
2. 7
3. (a) 0.3 (b) 2.9
4. 1
5. $\frac{12}{11}$
6. 0.75p
7.

x	10	20
$P(X=x)$	0.4	0.6

8. (a) 0.3 (b) 0.2
9. (a) 0.2 (b) 2.08
10. 2.75
11.

x	4	6	8	9	11	14
$P(X=x)$	0.16	0.32	0.16	0.16	0.16	0.04

   Loss of £1.20.

12. $£\frac{3}{8}(7+x)$ (a) 5 (b) Loss of £3.75
13. (a) 24
(c)

x	0	1	2	3	4
$P(X=x)$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	0	$\frac{1}{24}$

(d) 1
14. $\frac{11}{3}$
15. 2

Exercise 4c Expectation and variance (page 251)

1. (a) 2.3 (b) 5.9 (c) 0.61
2. (a) 0.35 (b) 4.2
3. (a) 1.45 (b) 2.45 (c) 12.15
4. (a) 3.5 (b) $15\frac{1}{8}$ (c) 14.5 (d) $2\frac{11}{12}$
5. (a) 2.56
6. (a) 3.5 (b) 14 (c) 5.5 (d) 84 (e) 1.75
7. (a) 2 (b) 3 or −3
8. (a) $\frac{1}{32}$, 1, $1\frac{31}{32}$
9. (b)

x	4	6	8	10	12
$P(X=x)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

(c) 4
10. (a) 4.2 (b) $7\frac{1}{3}$ (c) 3.67
11. (a) $\frac{1}{10}$ (b) $3\frac{1}{2}$ (c) $15\frac{7}{30}$ (d) $2\frac{59}{60}$ (e) $47\frac{11}{15}$
12. (a) $1\frac{2}{3}$ (b) $3\frac{1}{3}$ (c) $\frac{5}{9}$
13.

x	0	1	2
$P(X=x)$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$

(a) $\frac{8}{7}$ (b) $\frac{12}{7}$ (c) $\frac{20}{49}$ (d) $\frac{180}{49}$
14. (a) 5 (b) 2.5 (c) 10 (d) 10
15. 144
16. (a) $\frac{6}{7}$ (b) 0.639
17. $P(X=x)=\frac{10-x}{45}$, $x=1, 2, \ldots, 9$; $3\frac{2}{3}$, 2.21, 1;
$P(X=x)=(\frac{4}{5})^{x-1}(\frac{1}{5})$, $x=1, 2, \ldots$
18. (a) $\frac{1}{12}$ (b) 0 (c) 6 (d) 2.45
19. (a) 0.04 (b) 5 (c) 4 (d) 7 (e) 16
20. (a) Loss £3 (b) (i) $p=0.12$, $q=0.08$ (ii) 645, 8
21. (a) £2 (b) (i) 4 (ii) 17 (iii) 1

Exercise 4d Cumulative distribution function (page 255)

1.

y	0.1	0.2	0.3	0.4	0.5
$P(Y \leqslant y)$	0.05	0.3	0.6	0.75	1

2. (a) 0.41 (b) 0.87 (c) 0.46 (d) 0.13 (e) 2.58
3. (a)

x	0	1	2
$F(x)$	$\frac{25}{36}$	$\frac{33}{36}$	1

(b)

x	1	2	3	4	5	6
$F(x)$	$\frac{11}{36}$	$\frac{5}{9}$	$\frac{3}{4}$	$\frac{8}{9}$	$\frac{35}{36}$	1

(c)

x	0	1	2	3
$F(x)$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{7}{8}$	1

4.

x	3	4	5	6	7
$P(X=x)$	0.01	0.22	0.41	0.22	0.14

; 0.9724

5. (a) $\frac{4}{9}$ (b) $\frac{1}{3}$ (c) $P(X=x)=\frac{2x-1}{9}$, $x=1, 2, 3$ (d) $\frac{17}{9}$
6. (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $P(X=x)=\frac{1}{3}$, $x=1, 2, 3$ (d) 0.816
7. (b)

x	1	2	3	4
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{15}{64}$	$\frac{1}{64}$

(c) $2\frac{1}{64}$, 0.547 (d) $\frac{1}{4}$
8. (a) 0.9900 (b) 0.1746 (c) 0.5886
(d) 0.5565 (e) 0.9785

Exercise 4e Combinations of random variables (page 261)

1. (a) 26 (b) 15 (c) 17 (d) 59 (e) 59
2. (a) 0 or 12 or −12 (b) 294
3. (a) 1 (b) −1 (c) 34 (d) 14 (e) 14 (f) 30
4. (a) 1.3, 1, 1.01, 0.8
(b)

$x+y$	0	1	2
$P(X+Y=x+y)$	0.12	0.14	0.32

$x+y$	3	4	5
$P(X+Y=x+y)$	0.2	0.18	0.04

(c)

$x-y$	−2	−1	0
$P(X-Y=x-y)$	0.12	0.14	0.32

$x-y$	1	2	3
$P(X-Y=x-y)$	0.2	0.18	0.04

5. (a) 2.6, 0.24 (b) 5.2, 0.48 (c) 7.8, 0.72
6. $29\frac{1}{6}$
7. (a) 0.1 (b) 3 (c) 1 (d) 0.2 (e) 12 (f) 3

Miscellaneous exercise 4f (page 266)

1. 0.1825, £1.75
2. (a) $\frac{1}{13}$ (b) 2, $\frac{12}{13}$
3. (a) 0.01 (b) 3.54, 0.4684 (c) 14.7, 11.71
4. $\frac{120}{49}$, 2.57
5. $\frac{1}{28}$, 3.5, 1.25, 12, 20
6. (a)

x	1	2	4	5
$P(X=x)$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{1}{3}$	$\frac{1}{6}$

(b)

y	2	3	4	5	6	7	8	9	10
$P(Y=y)$	$\frac{1}{144}$	$\frac{5}{72}$	$\frac{25}{144}$	$\frac{1}{18}$	$\frac{11}{36}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{36}$

$6\frac{1}{6}$, $3\frac{35}{72}$
7. $\frac{1}{15}$, $\frac{2}{3}$, $\frac{34}{45}$, $\frac{1}{75}$, $\frac{4}{3}$, $\frac{68}{45}$
8. (a) $\frac{3}{8}$ (b)

d	0	1	2	3
$P(D=d)$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

(c) 1.25
9. (a) 0.1248 (b) 2.8352, 236
10. (b)

b	0	1	2	3	4
$P(B=b)$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{4}{27}$	$\frac{8}{81}$	$\frac{16}{81}$

(c) $1\frac{49}{81}$ (e) $\frac{16}{81}$
11.

y	0	1	2	3
$P(Y=y)$	0.3	0.34	0.2	0.16

(a) 1.22 (b) 1.0916 (c) 0.36

12.

x	2	3	4	5	6	7	8	9	10	11	12
$P(X=x)$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{4}{25}$	$\frac{4}{25}$	$\frac{3}{25}$	$\frac{2}{25}$	$\frac{3}{25}$	$\frac{2}{25}$	$\frac{1}{25}$

7.2, £75
13. (a) $\frac{1}{36}$ (b) $\frac{5}{36}$ (c) $\frac{11}{36}$; $-\frac{1}{36}$, 7
14. (a) $\frac{1}{8}$, $\frac{5}{24}$ (b) 2.78 (c) 0.260
15. (a) 0.8 (b) $-0.24p$ (c) $3.34p^2$
16. (a) 1.7, 1.18 (b) 4.76
17. (a) 1, $\frac{4}{5}$ (b) $\frac{3}{5}$, $\frac{6}{25}$ (c) 11.2, 7.28

t	0	1	2	3	4
$P(T=t)$	$\frac{2}{13}$	$\frac{9}{25}$	$\frac{8}{25}$	$\frac{11}{75}$	$\frac{1}{25}$

18. $P(X=x)=\frac{1}{6}$, $x=1, 2, 3, 4, 5$; $P(X=6)=0$; $P(X=x)=\frac{1}{36}$, $x=7, 8, \ldots, 12$; $4\frac{1}{12}$, $\frac{6}{17}$

Mixed test 4A (page 269)

1. (a) 0.2 (b) 8 (c) 11.6
2. (b)

x	0	1	2	3	4
$P(X=x)$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{5}{18}$	$\frac{1}{9}$	$\frac{1}{36}$

(c) $1\frac{1}{3}$ (e) 0, $\frac{20}{9}$
3. (a)

x	0	1	2	3	4	5
$P(X=x)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

(b) $1\frac{17}{18}$ (c) $\frac{4}{9}$

Mixed test 4B (page 269)

1. (a) 0.4 (b) 0.8 (c) 2.6 (d) 1.44 (e) 15.6
2. (a)

x	4	5	6	8	9	12
$P(X=x)$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{1}{36}$

(b) $6\frac{1}{3}$, $43\frac{13}{18}$, $3\frac{11}{18}$ (c) Loss of £1 (d) $\frac{2}{19}$
3. (a)

s	1	2	3	4	6	12
$P(S=s)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

(b) 4.5 (c) 11

Chapter 5

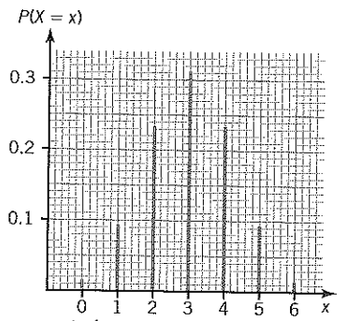
Exercise 5a The uniform and geometric distributions (page 276)

1. (a) 0.2 (b) 8 (c) 0.4
2. (a) 0.096 (b) 0.179 (c) 0.725 (d) 2.86
3. (a) 0.9744 (b) 0.01024 (c) 1 (d) $1\frac{2}{3}$ (c) 2.5
4. (a) 1 (b) 0.7599
5. (a) 0.0226 (b) 0.00374
6. (a) (i) 0.6 (ii) 0.3 (iii) 4.5 (iv) 2.87
(b) (i) 0.0531 (ii) 1 (iii) 10
7. (a) 1 (b) 2 (c) 1.41
8. (a) 0.128 (b) $X \sim \text{Geo}(0.2)$ (c) 0.512
9. (a) $P(X=4)=0.7^3 \times 0.3=0.1029$
(b) The first success is at the nth attempt.
(c) There are at least n attempts before the first success is obtained.
10. 0.7225
11. 0.00026
12. 2
13. (a) 0.0864 (b) 2.5 (c) 1.94 (d) 1 (e) 0.028
14. £1.75
15. 0.0047, December 22nd
16. (a) $\frac{1}{6}$ (b) $\frac{25}{216}$ (c) $\frac{125}{216}$ (d) 1 (e) 6 (f) 17

Exercise 5b The binomial distribution (page 285)

1. (a) 0.267 (b) 0.850
2. (a) 0.234 (b) 0.000107 (0.0001 from tables)
3. (a) 0.279 (b) 0.983 (c) 0.594
4. (a) 0.00549 (b) 0.157 (c) 0.503
5. 0.00200
6. (a) 0.318 (b) 0.671 (c) 0.647 (d) 0.0324
7. 0.344
8. 0.5
9. 0.3456
10. (a) (i) 0.0424 (ii) 0.623 (b) 12
11. 0.0963, improve with practice
12. (a) 0.0105 (b) 0.988 (c) 0.358
13. (a) 0.329 (b) 0.461
14.

x	$P(X=x)$
0	0.0156
1	0.0938
2	0.2344
3	0.3125
4	0.2343
5	0.0938
6	0.0156



symmetrical
15. 9
16. 68; not strictly binomial as p is not constant, but model can be used if there are a large number of bulbs in the box.
17. (a) 0.000416 (tables give 0.0004) (b) 0.0197
18. 5
19.

x	$P(X=x)$
0	0.0000
1	0.0001
2	0.0011
3	0.0109
4	0.0617
5	0.2096
6	0.3960
7	0.3206

20. 4
21. Experiment 1 – no, 3 outcomes; Experiment 2 – yes, constant probability of obtaining black (or white), independent trials; Experiment 3 – no, trials not independent.

Exercise 5c Expectation, variance and mode of the binomial distribution (page 290)

1. 2.5, 1.5
2. (a) 1.38 (b) 4
3. 8, 1.30
4. (a) 0.2 (b) 0.00551

5. (a) 0.25 (b) 2.5 (c) 0.282
6. 0.1, 0.23
7. (a) 10 (b) 0.000390
8. (a) 3 (b) 3 (c) 0.633
9. (a) 0.994 (b) 2
10. 0, 0, 3, 13, 30, 36, 18
11. 2500
12. 0.06; 293, 94, 12, 1, 0, 0
13. (a) 0.68 (b) 8, 1.6
14. (a) 0.25 (b) 1.5
15. 1, 0.894 (a) 5 (b) 0.2

Exercise 5d The Poisson distribution (page 297)

1. (a) 0.180 (b) 0.0527 (c) 0.195 (d) 0.670
2. (a) 0.983 (b) 0.184 (c) 0.199
3. (a) 0.0821 (b) 0.560 (c) 0.0631
4. (a) 0.603 (b) 0.616 (c) 0.00246
5. (a) 0.0821 (b) 0.242 (c) 0.759
   (d) 0.0486 (e) 0.125
6. (a) 0.191 (b) 0.0498 (c) 2.45
7. 0.371
8. (a) 0.0382 (b) 0.122
9. 0.677
10. (a) 3 (b) 0.145
11. (a) 90, 72, 29, 8, 1, 0 (b) 44, 44, 22, 8, 2
12. Random events; 0.5, 0.481; 31, 16, 4, 1, 0
13. (a) 0.261 (b) 6

Exercise 5e The Poisson approximation to the binomial (page 300)

1. (a) (i) 0.0476 (ii) 0.0498 (b) (i) 0.225 (ii) 0.224
   (c) (i) 0.171 (ii) 0.168
2. (a) (i) 0.184 (ii) 0.0190 (b) 0.271 (c) 0.0498
3. (a) 0.287 (b) 0.191
4. (a) $\frac{1}{36}$ (b) 0.713
5. (a) 0.647 (b) 0.185
6. 0.109, 185
7. (a) 0.677 (b) 0.017; 1498
8. (a) 0.468 (b) 0.703
9. 0.0150
10. (a) 0.47 (b) 0.041
    Poisson applies since $p < 0.1$ and $n = 50$. Events may not be independent. After mis-dialling, you are likely to be more careful.
11. Random sample, 0.305

Exercise 5f Sums of Poisson variables (page 303)

1. 0.121
2. (a) 0.189 (b) 0.308 (c) 0.184
3. (a) 0.323 (b) 0.119
4. (a) 0.301 (b) 0.080 (c) 0.251

Miscellaneous exercise 5g (page 307)

1. 0.752, 0.537
2. (a) 3 (b) 0.223 (c) 0.988
3. (a) 0.733 (b) 0.0703
4. (a) (i) 0.434 (ii) 0.378 (iii) 0.148 (iv) 0.0401
   (b) (i) 45 (ii) 111; $N > 20$
5. 0.507
6. (a) (i) 0.130 (ii) 0.271 (iii) 0.276; 65, 0.0159
   (b) 90, 3
7. (a) 0.270 (b) 0.350 (c) 0.182
   (d) 0.124 (e) £45
8. (a) 0.223 (b) 0.116 (c) 9.28, 2.86 (d) 18.9
   (e) Part (c) gives 223, part (d) gives 227, increase
9. (a) Large number of balls (b) 0.799
10. 0.790, calls occur randomly
11. (a) 0.104 (b) 0.283 (c) 0.00113 (d) 9
12. 0.632, 0.069, 0.154
13. (a) (i) $X \sim B(28, 0.004)$ (ii) 0.00545 (b) 0.785
    (c) independence
14. (a) 0.311 (b) 0.959; 3.6, 1.2
15. (a) 0.253 (b) 3.6, 1.59
16. (a) (i) 0.201 (ii) 0.00637 (b) 2 (c) 5, 2 (d) 14
17. (a) 0.203 (b) (ii) 0.136 (c) 0.316
    (d) Assume p constant; very unlikely in First World War
18. $P(X = x) = e^{-\lambda}\frac{\lambda^x}{x!}$, λ, λ
    (a) 0.082 (b) 0.242; 6.15
19. (a) 0.908 (b) 9
20. (a) 3, 7 (b) 20, 20
    Reason for (a) $E(Y - X) \neq \mathrm{Var}(Y - X)$
    Reason for (b) $2Y + 10$ could not take values less than 10.
21. 600 m, Po(2.5), 0.0821, 0.109, 0.779, 0.207
22. (a) (ii) 1.5 (b) 0.577 (c) 0.0249
23. 0.407, 0.366, 0.165, 0.0629, 0.816, 0.0518
24. (a) 22 (b) 19; 39
25. (a) 0.135 (b) 0.323; 0.81
26. (a) 0.387 (b) 0.929 (c) 0.893
    (d) 0.205 (e) 0.816; 0.0290
27. (a) 0.0902 (b) 0.0613; 4

Mixed test 5A (page 312)

1. (a) 0.159 (b) 0.766;
   Query independence: friends may have joint engagements.
2. (a) 0.152 (b) 0.567 (c) 0.285
3. (a) $X \sim B(150, \frac{1}{80})$, $\lambda = 1.875$, $p < 0.1$, $n > 50$
   (b) 0.559 (c) 369
4. (a) $X \sim \mathrm{Po}(0.6)$, X is number of boxes in a square km.
   (b) 0.549 (c) 0.0231
   (d) Probably not suitable; different scatter of telephone boxes in the city.
5. (a) 4.8, 0.98 (c) 0.737 (d) 0.388

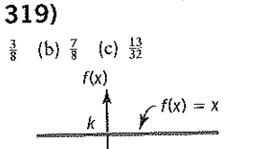
Mixed test 5B (page 313)

1. $\frac{2}{3}$ (a) $5(1-p)p^4$ (b) $10(1-p)^3p^2$
2. (a) $Y \sim \mathrm{Geo}(\frac{1}{6})$ (b) 30 (c) 0.233
3. (a) Binomial (b) Poisson (c) $e^{-\lambda}\frac{\lambda^3}{6}$
   (d) $1 - e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2}\right)$; 0.013, 0.014, 0.182
4. (a) 0.221 (b) 0.987
5. (a) 0.249 (b) 0.929 (c) 0.508; 0.542

Chapter 6

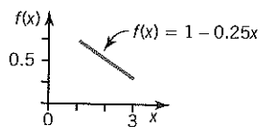
Exercise 6a Calculating probabilities (page 319)

1. (a) $\frac{3}{8}$ (b) $\frac{7}{8}$ (c) $\frac{13}{32}$
2. (a)

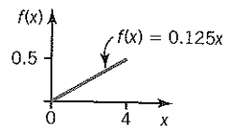


(b) 0.2 (c) 0.74

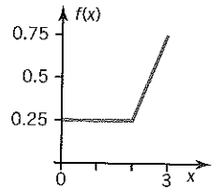
3. (a) 0.25
   (b)



   (c) 0.66
4. (a) $\frac{3}{56}$ (b) $\frac{19}{56}$, $\frac{37}{56}$
5. $c = 1$, $k = 4$
6. (a) 0.125 (b)



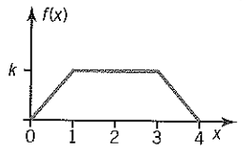
   (c) 0.328
7. (a) 0.25
   (b)



   (c) 0.25 (d) 0.3125 (e) 0.3475

Exercise 6b Expectation E(X) (page 323)

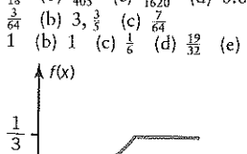
1. (a) $\frac{9}{16}$ (b) 1 (c) 2 (d) 1.6 (e) $2\frac{1}{24}$
2. (a)



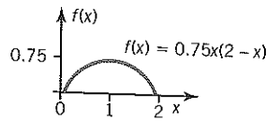
   (b) $\frac{1}{3}$ (c) 2
3. 3
4. 6 m
5. (a) $\frac{2}{75}$ (b) $\frac{70}{9}$ (c) 0.48, money bond
6. 2, 0.124
7. 2.5, 0.803, 0.456
8. (a) 2.875 kg (b) £4.75, $\frac{3}{16}$
9. (a) 0.4 (b) 2.6 (c) 1.5

Exercise 6c Standard deviation and variance (page 333)

1. (a) 1.5 (b) 2.4 (c) 0.15 (d) 0.387
2. (a) 0.5 (b) $2\frac{1}{3}$ (c) $2\frac{1}{12}$ (d) 1.44
3. (a) $1\frac{5}{6}$ (b) $3\frac{2}{3}$ (c) $\frac{11}{36}$ (d) 0.553
4. (a) $1\frac{3}{14}$ (b) $1\frac{27}{35}$ (c) $\frac{291}{980}$ (d) 0.545
5. (a) $\frac{4}{5}$ (b) $\frac{2}{3}$ (c) $\frac{2}{75}$ (d) 0.163
6. (a) $1\frac{19}{24}$ (b) $4\frac{1}{24}$ (c) $\frac{479}{576}$ (d) 0.912
7. (a) $\frac{5}{18}$ (b) $\frac{244}{405}$ (c) $\frac{731}{1620}$ (d) 0.672
8. (a) $\frac{3}{64}$ (b) 3, $\frac{3}{5}$ (c) $\frac{7}{64}$
9. (a) 1 (b) 1 (c) $\frac{1}{6}$ (d) $\frac{19}{32}$ (e) 1
10. (a)



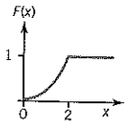
    (b) $3\frac{5}{12}$ (c) $12\frac{31}{45}$ (d) 1.008
11. (b) $2, 4 - \frac{4}{\ln 3}$
12. $a = 2$, $k = 0.75$;



    ; 0.2
13. 0.6, 0.2

Exercise 6d Cumulative distribution function (page 339)

1. (a) $F(x) = \begin{cases} \frac{x^3}{8} & 0 \leqslant x \leqslant 2 \\ 1 & x \geqslant 2 \end{cases}$



   (b) 1.59

2 (a) $F(x) = \begin{cases} \frac{1}{8}(8x - x^2 - 7) & 1 \leqslant x \leqslant 3 \\ 1 & x \geqslant 3 \end{cases}$ (b) $\frac{9}{32}$

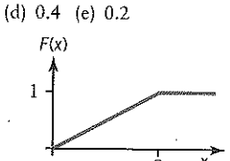
3. (a) $\frac{1}{5}$ (b) $F(x) = \begin{cases} \frac{1}{5}(x - 1) & 1 \leqslant x \leqslant 6 \\ 1 & x \geqslant 6 \end{cases}$ (c) 2 (d) 2.5

4. (a) $F(x) = \begin{cases} \frac{x}{4} & 0 \leqslant x \leqslant 2 \\ \frac{1}{4}(x^2 - 3x + 4) & 2 \leqslant x \leqslant 3 \\ 1 & x \geqslant 3 \end{cases}$ (b) 2

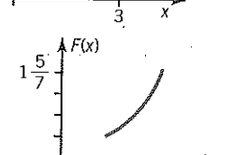
5. (a) 0.1215 (b) 0.841 (c) 0.880

6. (a) 1.5 (b) 0.75 (c) $F(x) = \begin{cases} \frac{1}{3}x & 0 \leqslant x \leqslant 3 \\ 1 & x \geqslant 3 \end{cases}$

   (d) 0.4 (e) 0.2



7. (a) $\frac{3}{7}$



   (b) 0.272 (c) $F(x) = \begin{cases} \frac{1}{7}(x^3 - 1) & 1 \leqslant x \leqslant 2 \\ 1 & x \geqslant 2 \end{cases}$ (d) 1.65

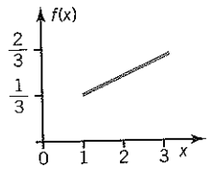
8. $\frac{3}{4}, \frac{19}{80}$, $F(x) = \begin{cases} \frac{3}{4}x - \frac{1}{16}x^3 & 0 \leqslant x \leqslant 2 \\ 1 & x \geqslant 2 \end{cases}$, 0.007

9. (a) $\frac{1}{3}, \frac{1}{3}$

   (b) $F(x) = \begin{cases} \frac{x^2}{6} - \frac{2x}{3} + \frac{2}{3} & 2 \leqslant x \leqslant 3 \\ \frac{x}{3} - \frac{5}{6} & 3 \leqslant x \leqslant 5 \\ 2x - \frac{x^2}{6} - 5 & 5 \leqslant x \leqslant 6 \\ 1 & x \geqslant 6 \end{cases}$

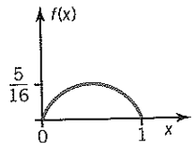
   (c) $\frac{1}{3}$ (d) $\frac{1}{24}$

10. (a)



(b) $2\frac{1}{9}$ (c) $F(x)=\begin{cases}\frac{1}{6}x+\frac{1}{12}x^2-\frac{1}{4} & 1\leqslant x\leqslant 3\\ 1 & x\geqslant 3\end{cases}$ (d) 2.16

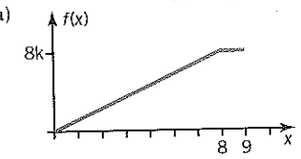
11. (a) 5 (b) $\frac{1}{6}$ (c) $\frac{5}{252}$; 543 tonnes



12. $F(x)=\begin{cases}\frac{1}{4}x & 0\leqslant x\leqslant 1\\ \frac{1}{5}+\frac{x^4}{20} & 1\leqslant x\leqslant 2\\ 1 & x\geqslant 2\end{cases}$ 1.565, 0.821

13. (b) 1, 2 (c) 0 (d) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

14. (a)



(c) $F(x)=\begin{cases}0.0125x^2 & 0\leqslant x\leqslant 8\\ 0.2x-0.8 & 8\leqslant x\leqslant 9\\ 1 & x\geqslant 9\end{cases}$

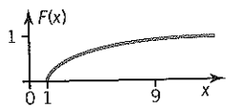
(d) 0.55

15. (a) 0.75 (b) 0.2

(c) $F(x)=\begin{cases}0.75x^2-0.25x^3 & 0\leqslant x\leqslant 2\\ 1 & x\geqslant 2\end{cases}$ (d) 0.288

16. (a) 0.455, 3 (b) 3.64, 4.95

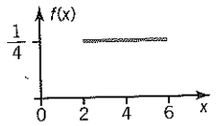
(c) $F(x)=\begin{cases}\frac{1}{\ln 9}\ln x & 1\leqslant x\leqslant 9\\ 1 & x\geqslant 9\end{cases}$



Exercise 6e Obtaining $f(x)$ from $F(x)$ (page 343)

1. (a) $f(x)=\frac{1}{4}$, $2\leqslant x\leqslant 6$

(b)

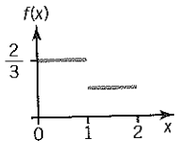


(c) 4 (d) 2

2. (a) 0.794 (b) 0.75

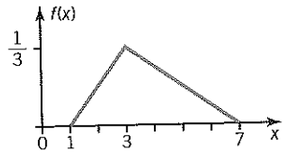
3. (a) 0.25 (b) $f(x)=1-0.5x$, $0\leqslant x\leqslant 2$
(c) 0.586 (d) $\frac{2}{9}$

4. (a) $\frac{1}{3}$ (b) $f(x)=\begin{cases}\frac{2}{3} & 0\leqslant x<1\\ \frac{1}{3} & 1\leqslant x\leqslant 2\\ 0 & \text{otherwise}\end{cases}$



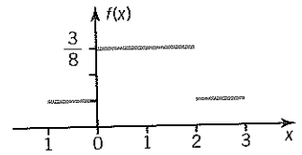
(c) $\frac{5}{6}$ (d) 0.553

5. (a) $f(x)=\begin{cases}\frac{1}{6}(x-1) & 1\leqslant x\leqslant 3\\ \frac{1}{12}(7-x) & 3\leqslant x\leqslant 7\\ 0 & \text{otherwise}\end{cases}$



(b) $3\frac{2}{3}$ (c) $1\frac{5}{9}$ (d) 3.45 (e) 0.595

6. (a)



(b) 1, $\frac{5}{6}$ (c) 0.25

7. (a) 1, $-\frac{1}{27}$ (b) $F(x)=\begin{cases}\frac{1}{27}x^3 & 0\leqslant x\leqslant 3\\ 1 & x\geqslant 3\end{cases}$

(c) $f(x)=\frac{1}{9}x^2$, $0\leqslant x\leqslant 3$

8. (a) 2 (b) $f(x)=2$, $0\leqslant x\leqslant 0.5$ (c) 0.25 (d) 0.144

Exercise 6f Uniform distribution (page 349)

1. (a) $\frac{1}{3}$ (b) 4.5 (c) 0.75 (d) $\frac{1}{3}$
2. (a) 0.5 (b) −3.5 (c) 0.866
3. (a) 5 (b) 0.325 (c) 3 (d) $1\frac{1}{3}$
4. 0.4
5. 0.577
6. (a) 4.5 (b) $2\frac{1}{12}$
7. (a) $a=3$, $b=11$ (b) 0.125

(c) $F(x)=\begin{cases}\frac{1}{8}(x-3) & 3\leqslant x\leqslant 11\\ 1 & x\geqslant 11\end{cases}$

8. (a) $f(x)=0.2$, $-2\leqslant x\leqslant 3$ (b) 1.44 (c) 2.5 (d) −1

Miscellaneous exercise 6g (page 355)

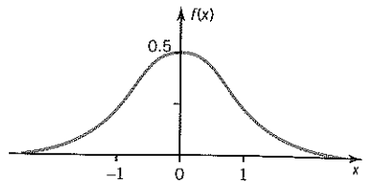
1. (a) $1\frac{1}{3}$ (b) $6\frac{2}{3}$
2. (a) 2.4 (b) 20, $\frac{1}{3}$, 0.178
3. (a) $\frac{2}{3}$ (b) $f(x)=\begin{cases}\frac{2}{3}x & 0\leqslant x\leqslant 1\\ 1-\frac{1}{3}x & 1\leqslant x\leqslant 3\\ 0 & \text{otherwise}\end{cases}$

$1\frac{1}{3}$, $\frac{7}{18}$

(c) 1.27; 0.875

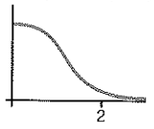
4. 0.8, 0.16, £8

5. (b)

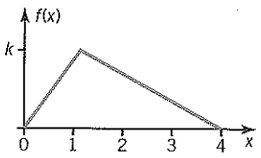


(c) $F(x)=\begin{cases}-\frac{1}{12x^3} & x\leqslant -1\\ \frac{1}{2}+\frac{1}{2}x-\frac{1}{12}x^3 & -1\leqslant x\leqslant 1\\ 1-\frac{1}{12x^3} & x\geqslant 1\end{cases}$ (d) 0, $\frac{11}{15}$

6. (a) −0.1875 (b) 0.2375 (d) 2
7. (b) 2.2 (c) 1.71 (d) 0.264 (e) 0.3645
8. (b) 0.5 (d) 0.36
9. (b) 30 hrs (d) $\frac{65}{81}$ (e) 0.0390
(f) The model does not allow for lifetimes over 90 hours.
10. (a) 3.8 hrs, 0.36 hrs^2 (b) 4 hrs (c) Approx 60%
11. (a) 0 (b) 0.15625 (c) (i) symmetry (ii) 0.05
(d) The player might make a similar mistake each time, resulting in more hits above the line than below, or vice versa.
(e) The range would reduce.
12. (b) 75 hours (c) $\frac{5}{16}$
(d) The model does not allow for $P(X>2.5)>0$, since $P(X>2)=0$
(e) Change to exponential model for $x>1.8$, say



13. (a)

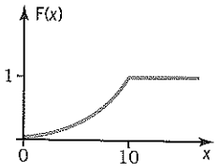


(c) $F(x)=\begin{cases}\frac{1}{4}x^2 & 0\leqslant x\leqslant 1\\ \frac{2}{3}x-\frac{1}{12}x^2-\frac{1}{3} & 1\leqslant x\leqslant 4\\ 1 & x\geqslant 4\end{cases}$

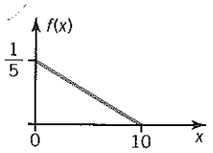
(d) £283.33 (e) $\frac{1}{3}$

14. 8, $\frac{1}{9}$, 39 litres

15. (a) 2.93 (b) $F(x)=\begin{cases}1-0.01(x-10)^2 & 0\leqslant x\leqslant 10\\ 1 & x\geqslant 10\end{cases}$



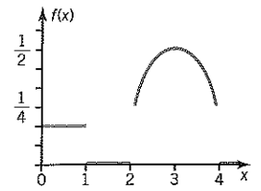
(c) $f(x)=\frac{1}{5}-\frac{1}{50}x$, $0\leqslant x\leqslant 10$



0.25

16. (b) $F(x)=\begin{cases}\frac{3}{17}x^2 & 0\leqslant x\leqslant 1\\ \frac{1}{17}(1+2x^3) & 1\leqslant x\leqslant 2\\ 1 & x\geqslant 2\end{cases}$ (c) 1.55 (d) 0.89

17. $\lambda=\frac{1}{3}$



$2\frac{7}{12}$, $\frac{79}{96}$

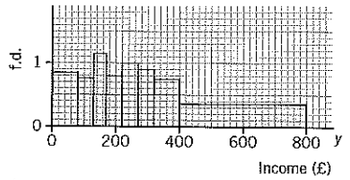
18. (a) $\frac{1}{3}$ (b) $f(x)=\begin{cases}\alpha & -1\leqslant x<0\\ 2\alpha & 0\leqslant x<1\\ 0 & x\geqslant 1, x<-1\end{cases}$

(c) $\frac{1}{6}$ (d) 0.553 (e) $\frac{11}{18}$

19. (a) 2.1, 1.29 (b) 1, 0.5

Mixed test 6A (page 358)

1. (a) f.d. 0.85, 0.76, 1.15, 0.8, 1, 0.9, 0.75, 0.36
Histogram to show incomes



(b) $\frac{1}{16}$ (c) 120
(d) From original data, 106 have income in this range. In the model, $f(x)=3k$, $0\leqslant x\leqslant 4$ gives too high an estimate; perhaps $f(x)=2.5k$, $0\leqslant x\leqslant 4$ would be better.

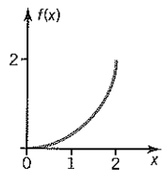
2. 4, $\frac{8}{15}$, $\frac{11}{225}$, 0.541

3. (a) $F(w)=\begin{cases}\frac{w^4}{5^5}(25-4w) & 0\leqslant w\leqslant 5\\ 1 & w\geqslant 5\end{cases}$

(b) 0.650 (c) 0.794 (d) 3.75 (f) Negatively skewed

Mixed test 6B (page 359)

1. (a)

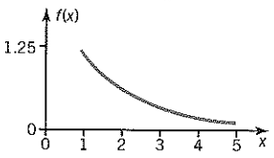


(b) 1.6 (c) 0.327 (d) $F(m)=0.5$, $F(\mu)<0.5$ $\therefore$ $m>\mu$

2. (b) $\frac{8}{15}$ (c) 0.577

3. (b) $1.25\left(1-\frac{1}{m}\right)=0.5$, $m=1\frac{2}{3}$

(c) 0.495 (d) $f(x)=\frac{1.25}{x^2}$, $1\leqslant x\leqslant 5$



Chapter 7

Exercise 7a Finding probabilities, where $Z \sim N(0, 1)$ (page 367)

1. (a) 0.8089 (b) 0.8089 (c) 0.1911 (d) 0.1911
2. (a) 0.0359 (b) 0.2578 (c) 0.9931 (d) 0.9131
   (e) 0.0049 (f) 0.9911 (g) 0.9686 (h) 0.2343
   (i) 0.0312 (j) 0.9484 (k) 0.9803 (l) 0.0021
3. (a) 0.05 (b) 0.05 (c) 0.0999 (d) 0.025 (e) 0.005
   (f) 0.01 (g) 0.0025 (h) 0.075
4. (a) 0.044 (b) 0.8185 (c) 0.1336 (d) 0.3023
5. (a) 0.1703 (b) 0.5481 (c) 0.3639 (d) 0.4582
   (e) 0.4798 (f) 0.9624 (g) 0.0337 (h) 0.9082
   (i) 0.2729 (j) 0.030 (k) 0.925 (l) 0.4508
   (m) 0.9 (n) 0.02
6. 50%
7. (a) 0.9 (b) 0.7
8. (a) 0.55 (b) 0.15
9. (a) 0.9 (b) 0.1

Exercise 7b Finding probabilities using $X \sim N(\mu, \sigma^2)$ (page 370)

1. (a) 0.0668 (b) 0.4013 (c) 0.1747
2. (a) 0.7054 (b) 0.0618 (c) 0.4621 (d) 0.00456
3. (a) 0.0548 (b) 0.1448 (c) 0.9544
4. (a) 0.0106 (b) 0.9857
5. (a) 0.3015 (b) 0.5231 (c) 0.3792
6. 740
7. 0.00003844
8. (a) 0.6554 (b) 8
9. (a) 0.0478 (b) 0.000817
10. (a) 0.9544 (b) 0.5784 (c) 0.0435
11. (a) 0.1056 (b) 0.7734 (c) 0.6678
12. 0.159, 0.775, 0.067, £37.56
13. 0.785, 0.397
14. 0.957

Exercise 7c Using the standard normal tables in reverse (page 376)

1. (a) 0.015 (b) 0.796 (c) −1.887 (d) −0.454
   (e) −0.562 (f) 1.019 (g) 0.842
2. (a) 1.94 (b) −0.695 (c) −0.915 (d) 0.722
3. (a) 0.91 (b) 1.66 (c) 0.674 (d) 2.05
4. 0.674, −0.674; 0.524
5. (a) 70 (b) 4.65 (c) 190.742 (d) 1.468
6. (458.92, 546.52)
7. (a) 0.6247 (b) 629.52 g (c) 3
8. 8, 1.158, (6.10, 9.90)
9. (a) (384.32, 415.68) (b) (394.608, 405.392)
10. (a) 0.9332 (b) 0.383; 106.6, 137
11. (a) 0.0548 (b) 26 (c) 67.4 (d) 2183
12. (a) 37.8% (b) (125.5, 194.5) (c) 0.405

Exercise 7d Finding μ or σ or both, where $X \sim N(\mu, \sigma^2)$ (page 381)

1. 30
2. 10.7
3. 8.31, 35.9%
4. 35.5
5. 1.75
6. 52.73, 11.96
7. 2.74, 2.78
8. (a) 6.99, 0.324 (b) 0.0105
9. 39.5, 5.32
10. 53.87, 16.48
11. 0.203
12. 92.7%, 1.32, 1.7%
13. 4.299 g
14. 4.46
15. 2080, 236
16. (a) 0.4875 (b) 281, 5.00
17. 5.2007, 0.00346; 0.0269
18. (a) 0.1587 (b) 128.4 (c) 1.31
19. 0.0401 (a) 0.459 (b) 0.003
20. 490 g, 12.2 g
21. (a) 19.50 (b) not symmetrical (c) 32

Exercise 7e Continuity corrections (page 386)

1. P(2.5 < X < 9.5)
2. P(3.5 < X < 8.5)
3. P(10.5 < X < 24.5)
4. P(1.5 < X < 7.5)
5. P(X > 54.5)
6. P(X > 75.5)
7. P(45.5 < X < 66.5)
8. P(X < 108.5)
9. P(X < 45.5)
10. P(55.5 < X < 56.5)
11. P(400.5 < X < 560.5)
12. P(66.5 < X < 67.5)
13. P(X > 59.5)
14. P(99.5 < X < 100.5)
15. P(33.5 < X < 42.5)
16. P(6.5 < X < 7.5)
17. P(X > 508.5)
18. P(X < 6.5)
19. P(26.5 < X < 28.5)
20. P(52.5 < X < 53.5)

Exercise 7f The normal approximation to the binomial (page 389)

1. 0.1958
2. (a) $np > 5$, $nq > 5$ (b) 0.0197 (c) 0.0968
3. (a) 0.0154 (b) 0.8145 (c) 0.02
4. (a) 0.657 (b) 0.2142
5. (a) 0.0318 (b) 0.8345
6. (a) 0.9474 (b) 0.6325 (c) 0.5914 (d) 0.0111
7. (a) 0.4502 (b) 0.0996 (c) 0.484
8. 20, 16, 0.00436
9. $P(R = r) = {}^nC_r(1-p)^{n-r}p^r$, np, $np(1-p)$
   (a) 0.2304 (b) 0.9222; 0.8531
10. 0.1432
11. 0.6886
12. $np > 5$, $nq > 5$ (a) 0.1853 (b) 0.1838 (c) 0.81%

Exercise 7g The normal approximation to the Poisson distribution (page 390)

1. (a) 0.6201 (b) 0.39 (c) 0.5406
2. (a) 0.3998 (b) 0.2004 (c) 0.3661 (d) 0.0637
3. (a) 0.313 (b) 0.5078 (c) 0.8335 (d) 0.1101
4. (a) 0.2614 (b) 0.2343 (c) 0.0558
5. 0.8901
6. 0.6887, 4
7. (a) 0.4574 (b) 0.173 (c) 0.8312
8. (a) 0.4594 (b) 0.5363
9. (a) (i) 0.9815 (ii) 0.3486 (iii) 0.9244 (b) 0.0094
10. (a) 0.199 (b) 0.185; 0.870
11. (a) 0.927 (b) 0.0102; 0.297

12. (a) Weevils are randomly scattered in the grain, the grain is selected at random.
    (b) (i) 0.950 (ii) 0.105 (c) 0.158
13. (a) 0.953 (b) 0.745 (c) 0.19
14. (b) 0.133 (c) 11 (d) 0.7119

Miscellaneous exercise 7h (page 398)

1. (a) 46.5% (b) 0.532 m (c) 1.00 M
2. (b) 0.0693 (c) 0.0746
3. (b) 11.5%
4. 50.154, 4
5. (a) (i) 0.0062 (ii) 0.5598 (b) 7.49 m (c) 0.27
   (d) Brian, since $P(X \geqslant 8) = 0.0207$ whereas for Alan $P(X \geqslant 8) = 0.0062$.
6. (a) 0.886
   (b) Data not symmetric but showing a positive skew.
7. (a) 1.2 (b) 53.6 (c) 54.2; 0.066
8. (a) (i) 4.95% (ii) 0, I, II
   (b) (i) 105.3 (ii) 106.45; 106.45
   (c) (i) 103.3, 3.98
   (ii) needs overhaul, standard deviation too high.
9. (a) 14.25 p (b) 736 g (c) 462 g
10. (a) (i) 0.250 (ii) 0.758 (iii) 0.00240 (b) 0.0433
11. (a) (i) 0.197 (ii) 0.820 (b) (ii) 19 (c) 0.2142
12. 0.360, 0.734
13. (a) 0.653 (b) 0.2224
14. (a) (i) 104 (ii) 33 (iii) 33 (b) 1000, 200
15. (a) 0.3154 (b) 0.3068; worse, 0.5245
16. 979.27, 17.27, 133
17. (a) random events, mean ≈ variance
    (b) 0.224 (c) 0.586 (e) 0.6201
18. (a) 0.988 (b) 0.855
    (c) 0.783 (Poisson), 0.784 (binomial)
19. (a) 0.649 (b) 0.965 (c) 0.371
20. (a) 0.988 (b) 0.624 (c) 0.828
21. (a) $np > 5$, $nq > 5$, $X \sim N(np, npq)$
    (b) $p < 0.1$, $n > 50$, $X \sim Po(np)$; 0.859
    (c) 0.204 (d) 0.034

Mixed test 7A (page 401)

1. (a) 29% (b) 402.62 ng/ml
2. (a) 25 (b) 0.673
3. (b) 0.0113 (c) 0.86
4. (a) 0.0548 (c) 0.356

Mixed test 7B (page 402)

1. Luxibrite, 0.936
2. (a) 0.1056 (b) 0.8641 (c) 815.68
3. (a) (i) 0.8944 (ii) 0.4931
   (b) only able to stay for a maximum of 60 minutes
   (c) mean + 3σ gives 6.55 pm
4. (a) 7.5 (b) randomly scattered (d) 0.901 (e) 0.2627
5. (a) (i) 0.0808 (ii) 0.1935
   (b) 0.295 (c) 0.0598

Chapter 8

Exercise 8a Sums and differences of normal variables (page 409)

1. (a) 210, 625 (b) $X \sim N(210, 625)$
   (c) 0.6554 (d) 0.7698
2. (a) 0.1319 (b) 0.0127
3. (b) 0.9324
4. 0.0745
5. (a) 0.5 (b) 0.8849 (c) 0.2779
6. (a) 0.0207 (b) (i) 0.0289 (ii) 0.0200 (iii) 0.6252
7. (a) 0.1247 (b) 0.6957
8. (a) 0.6298 (b) 0.1056
9. (a) 0.1728 (b) 0.6127 (c) 0.5
10. 0.2575
11. 0.1103, 0.753
12. 9.6, 0.522; (a) 1.8% (b) 22.2%
13. (a) (94.4, 105.6) (b) 92.55% (c) 22.14%
14. (a) 0.0787 (b) 3.02×10^{-6}

Exercise 8b Multiples of normal variables (page 413)

1. (a) 0.8962 (b) 0.9386
2. (a) 0.2398 (b) 0.2523
3. (a) 0.244 (b) 0.659 (c) 0.409
4. (a) 6, $\sqrt{2}$ (b) 0.2074 (c) 0.7601 (d) 0.5143
5. 0.2762
6. (a) 0.3446 (b) 0.6915; 0.0033, 0.304

Miscellaneous exercise 8c (page 417)

1. (a) 0.60 (b) 0.20 (c) 0.95 (d) 0.5
2. (a) 0.051 (b) 0.00155 (c) 0.9782
3. 1000, 172, 3000, 298, 0.16, 0.02
4. (a) 0.0888 (b) 0.6611
5. 0.0625, 0.2574, 0.5, 0.7123
6. (a) 0.0139 (b) 0.1587 (c) 0.9332
7. (a) 0.159 (c) 0.584
8. 12 kg, 57.0 g, 3.97%, 765 g
9. (a) (i) 0.1056 (ii) 0.8882 (b) 1028 g (c) 0.0537
10. (a) (i) 0.1056 (ii) 0.144 (b) 0.0188
11. (a) 0.1416 (b) 0.5999 (c) 14.96 m (d) 0.3043
12. (a) 0.798 (b) 0.323 (c) 0.132 (d) 0.228
13. (a) 0.252 (b) 0.0581 (c) 0.104

Mixed test 8A (page 419)

1. (a) $S \sim N(600, 105.8)$, 0.0724 (b) 0.8392
   (c) 0.1606 (d) 30.54 g
2. (a) 0.733 (b) 0.984
3. (b) 0.0802 (c) 0.6729

Mixed test 8B (page 420)

1. (a) 0.127 (b) (i) 0.0016 (ii) 0 (c) 0.1003
2. (a) 0.8413 (b) 0.5 (c) 0.4207; 0.9938
3. 0.84

Chapter 9

Exercise 9a Sampling methods (page 430)

2. (a) 6, 6, 6, 6, 6, 5, 5
4. (b) large : medium : small = 15 : 25 : 20

Exercise 9b Simulating random samples from given distributions (page 435)

Some answers depend on the random numbers used and on the method of allocation. These are possible answers.

10. (a) 1, 1, 1, 0, 3 (b) 4
11. 33.134, 33.193, 28.712
12. (a) 3, 5 (b) 1, 5 (c) 1007.2, 1016.8
13. 1.52
14. means of sample means ≈ distribution mean; variance of sample means ≈ $\frac{1}{2}$ variance of distribution
15. (a) 4 (b) 6.1826

Exercise 9c The distribution of the sample mean, $\bar{X}$ (page 443)

1. 0.0176
2. (a) 0.6234 (b) Approx. 4
3. (a) 0.1056 (b) 0.3092
4. (a) $\bar{X} \sim N\left(4.8, \frac{2.88}{50}\right)$ (b) 0.7975
5. (a) 8 (b) no
6. (a) 0.2399 (b) 0.0787 (c) 0.0127 (d) $n = 109$
7. 0.9212
8. 62
9. (a) 42 (b) 60
10. 5
11. 20, 3
12. (a) 12 (b) 20
13. 20500, 1768; no
14. 0.332, 0.0587, 0.009
15. 0.4948, 0.4944, 0.1211
16. (a) $P(X = 0) = \frac{1}{2}$, $P(X = 1) = \frac{1}{3}$, $P(X = 2) = \frac{1}{6}$
    (b) $\frac{2}{3}$ (c) 0.159

Exercise 9d Distribution of sample proportions (large samples) (page 447)

1. (a) 0.0745 (b) 0.0037
2. (a) 0.0057 (b) 0.527 (c) 0.1265
3. 0.0471
4. (a) 0.0648 (b) 0.0970
5. 0.7181
6. (a) 0.0648 (b) 0.0851 (c) 0.3068
7. (a) 0.22

Exercise 9e Point estimates and confidence intervals for μ (page 460)

1. 236, 7.58
2. (a) 48.875, 6.98 (b) 1.69, 8×10^{-6} (1 s.f.)
   (c) 22.79, 1.81
   (d) 15, 43.14 (e) 10, 3.11 (f) 9.71, 621.12
3. 0.5, 1.428
4. 205.16, 9.223
5. (a) (139.16, 140.5) (b) random sample
6. (a) (10.75, 14.15) (b) 3.4
7. (a) (448.7, 467.3)
   (b) The probability that this interval includes μ is 0.99.
   (c) No, z value less
8. (a) (79.19, 84.81) (b) (78.89, 85.11)
   (c) No, the central limit theorem can be used, since n is large.
9. (68.0, 70.0), random sample, central limit theorem can be applied.
10. (a) 3.612 (b) (747.3 g, 748.7 g)
    (c) random sample, central limit theorem can be applied.
11. (a) (1011, 1114) (b) 36
12. 28
13. (a) 5.06 g (b) 89%
14. Histogram: frequency densities 1.2, 3.6, 6.4, 11.4, 20.4, 10.2, 5, 1.8; 91.32, 7.42, 0.43, (90.5, 92.2)
15. 25.3, 3.6, (24.9, 25.8)
16. Histogram: frequency densities 0.8, 0.48, 0.3, 0.18, 0.1, 0.05, 0.04, 0.03, 0.02; 194, 176, (173.5, 214.5)

Exercise 9f Confidence intervals – small samples (t – distribution) (page 468)

1. (a) (177.21 cm, 182.12 cm) (b) 4.91 cm
2. (a) (3.59, 4.68) (b) 0.146
3. (8.07, 9.13)
4. (32.08, 33.22), 380
5. (a) 5.13 (b) 0.588 (c) (4.70, 5.56)
6. (14.98 g, 15.78 g)
7. (9.804, 9.808)

Exercise 9g Confidence intervals for p (page 471)

1. (a) (0.622, 0.738)
   (b) The normal approximation to the binomial has been used in the underlying distribution.
2. (a) (0.293, 0.427) (b) (0.273, 0.447)
3. (a) (0.238, 0.362) (b) 90
4. (a) 0.28 (b) (0.176, 0.384)
5. (0.156, 0.344)
6. (a) (0.223, 0.352) (b) wider
7. (a) Random sample (0.244, 0.283) (ii) 90 approximately
   (b) (i) 0.26
8. (a) (0.351, 0.369) (b) 5277
9. (0.509, 0.547)

Miscellaneous exercise 9h (page 478)

1. (124.34, 125.60), 4
2. (£93.59, £101.48)
3. 1.13, 0.0603, ($1.07, £1.19)
4. 9.71, (172.3, 173.3)
5. (a) 3, (2.04, 3.96) (b) 30%, (25.2%, 34.8%)
6. 0.059, 0.61
7. (a) Lifetime of bulb follows a normal distribution; the items in the box constitute a random sample.
   (b) (1774 hours, 1798 hours)
8. (a) 268 (b) smaller, critical z value less
9. (0.139, 0.315); there is a 1% chance that the interval has not trapped μ.
10. (a) 26.525, 1.24 (b) (26.20, 26.85) (c) justified
    (d) n large, use Central Limit theorem
11. (a) (28.98 cm, 29.42 cm) (b) Large sample
    (c) $\bar{X}$ normally distributed, random sample
    (d) (26.78 cm, 31.62 cm)
    (e) no; 30.5 out of range of 95% confidence interval for μ
12. (92.32, 99.68)
13. (a) (202.4, 207.4) (b) 0.2, (0.057, 0.343)
14. (0.123, 0.392), (170.84, 178.16), (165.57, 186.83)
15. 25.35, 0.13, (25.15, 25.6), valid
16. (a) (0.303, 0.357)
    (c) 10% probability that interval did not trap μ; people changed their minds at the last minute
17. (£35.60, £130.80)
18. (35.03 mg, 35.31 mg)
19. (13.10 mm, 14.72 mm)
20. (47.02 cm, 51.38 cm)
21. (0.0825 mm, 0.242 mm)

Mixed test 9A (page 481)

1. (a) 0.391 (b) 93%
2. 14
3. (0.23, 0.35); the normal approximation to the binomial has been used in the underlying theory; only cars in the car park were sampled which may not constitute a random sample.
4. (18.51, 19.49)

Mixed test 9B (page 482)

1. (b) (92.01, 93.19)
   (c) Central Limit theorem can be applied.

2. (0.35, 0.49), 0.14
3. (a) $\left(\bar{x} - \frac{38.64}{\sqrt{n}}, \bar{x} + \frac{38.64}{\sqrt{n}}\right)$ (b) 6000
4. (a) (244.2 g, 250.2 g) (b) 6.0 g (c) smaller

Chapter 10

Exercise 10a Testing p in a binomial distribution (small samples) (page 494)

1. H_0: $p = 0.7$, H_1: $p > 0.7$; no evidence
2. (a) H_0: $p = 1/6$, H_1: $p > 1/6$
   (b) There is no evidence that die is biased in favour of 4.
3. (a) Do not reject H_0 (b) Reject H_0
4. (a) Evidence to suggest decrease.
   (b) No evidence to suggest decrease.
5. (a) $x \geqslant 5$
   (b) The probability that H_0 is rejected when it is in fact true. (c) 0.1
6. (a) Accept H_0 (b) Reject H_0 (c) Reject H_0
   (d) Accept H_0 (e) Accept H_0 (f) Reject H_0
   (g) Reject H_0 (h) Accept H_0
7. (a) Driving instructor is over-estimating pass rate.
   (b) $x \geqslant 3$
8. She could have been guessing.
9. (a) $x \leqslant 2$ (b) 0.803
10. (a) 15% (b) 0.15(09) (c) 28% (2 s.f.)
11. (a) 7.5% (2 s.f.)
    (b) same as significance level
    (c) 66% (2 s.f.)

Exercise 10b Testing λ in a Poisson distribution (page 500)

1. Increased
2. (a) Not increased (b) Decreased
3. H_0: $\lambda = 9$, H_1: $\lambda > 9$, not increased
4. (a) 0.0424 (b) 0.849
5. (a) Accept H_0 (b) Accept H_0 (c) Accept H_0
   (d) Reject H_0 (e) Accept H_0 (f) Reject H_0
6. H_0: $\lambda = 3.5$, H_1: $\lambda > 3.5$, not increased

Miscellaneous exercise 10c Binomial and Poisson tests (page 504)

1. (a) 0.028 (b) 0.131
   (c) 0.261; H_0: $p = 0.6$, H_1: $p > 0.6$; teacher is not underestimating
2. (a) 0.552 (b) 6, 0.296
   (c) The probability he scores a penalty kick remains constant at 0.7.
   (d) H_0: $p = 0.7$, H_1: $p > 0.7$
   (e) No evidence of improvement (f) strengthened
3. Manufacturer's claim is not accepted; discrete distribution, $P(X \leqslant 12) = 3.6\%$, $P(X \leqslant 13) = 17.1\%$.
4. (a) H_0: $p = 0.2$, H_1: $p > 0.2$
   (b) $X \sim B(25, 0.2)$ (c) 9
5. (a) (i) 0.0278 (ii) 0.0384 (iii) 0.0768
   (b) H_0: $p = 0.5$, H_1: $p \neq 0.5$, no indication of whether looking for evidence of more males or more females.
   (c) Evidence of more males than females, $x \geqslant 13$
6. (a) 37% (b) 42%
   (c) (i) The consumer group has used a high value for the significance.
   (ii) Choose 5% or 10% significance level to maintain credibility.
7. (a) 2, 1.18 (b) 0.302
   (c) H_0: $p = 0.2$, H_1: $p < 0.2$, not reduced
   (d) reduced
8. (a) 0.430 (b) 0.962 (c) 0.00459
   (d) H_0: $p = 0.9$, H_1: $p < 0.9$, looking for a decrease
   (e) No evidence that service has deteriorated.
   (f) $x \leqslant 12$; $P(X \leqslant 12) < 0.05$, whereas $P(X \leqslant 13) > 0.05$
9. (a) Defects occur randomly and independently, with no two defects at the same spot.
   (b) (i) 0.209 (ii) 0.221
   (c) 0.140
   (d) H_0: $\lambda = 2.4$, H_1: $\lambda > 2.4$, evidence that number of defects has increased.
10. (a) (i) 0.181 (ii) 0.999 (b) 0.018
    (c) No evidence of decrease.
11. 0.0057, 9 mins, not significant

Mixed test 10A (Binomial) (page 506)

1. 0, 1, 9, 10
2. (a) H_0: $p = 0.15$, H_1: $p < 0.15$, evidence that new procedure has been successful.
   (b) Staff making an effort during the first week, take sample over a longer period of time.
3. No evidence to support gardener's claim.

Mixed test 10B (Poisson) (page 506)

1. (a) Poisson, 2.1
   (b) (i) 0.650 (ii) 0.222
   (c) Evidence suggests higher rate.
2. (a) (i) 0.138 (ii) 0.847
   (b) H_0: $\lambda = 7.5$, H_1: $\lambda < 7.5$, does not provide significant evidence.
3. (a) Nominally 5% (between 4.26% and 8.39%)
   (b) 76% (2 s.f.).

Chapter 11

Exercise 11a z-tests for a normal population or large sample size (page 522)

1. (a) $z = -1.095$, accept H_0 (b) $z = 1.845$, reject H_0
   (c) $z = 2.5$, reject H_0 (d) $z = -2.778$, reject H_0
2. $z = -0.943$, no
3. It could be 103.5
4. $z = 2.487$, yes
5. $z = 1.909$, distribution of the sample mean is approximately normal.
6. $z = 1.987$, no evidence
7. (a) $\bar{x} < 91.5065$ minutes (b) 0.0093 (c) 0.3286
8. $z = 0.983$, accept mean is zero
9. $5.778 < \bar{x} < 6.222$
10. (a) $z = 1.778$, accept H_0 (b) $z = 1.778$, reject H_0
    (c) $z = -1.428$, reject H_0 (d) $z = -2.487$ accept H_0
11. (a) Reject H_0 and conclude mean is not 52. (b) 0.04
12. (a) 0.0817 (b) 0.665
13. $z = 2.946$, yes
14. (b) 0.24 (c) $\mu > 389.7$ (d) 0.0494

Exercise 11b t-tests for a normal population, small sample size (page 527)

1. (a) $t = 0.909$, accept H_0 (b) $t = -1.89$, accept H_0
   (c) $t = 2.15$, reject H_0 (d) $t = -3.07$, accept H_0
2. $t = 2.828$, evidence of improved times
3. (a) $t = -3.54$, underweight (b) $z = -3.2$, underweight

4. $t = -1.1$, no
5. $t = 2.284$, mean greater than 4.3
6. (a) $t = -3.23$, no (b) (1.69, 2.88)
7. (a) $z = -1.66$, no change in mean
   (b) 0.324, $t = -2.33$, change in mean
8. X is normally distributed, $t = 1.80$, accept null hypothesis
9. $H_0: \mu = 27$, $H_1: \mu \neq 27$, $t = 2.9$, mean is 27
10. $H_0: \mu = 50$, $H_1: \mu < 50$, $t = -0.435$, not overstating

Exercise 11c Testing a binomial proportion large n (page 532)

1. (a) $z = 1.59$, accept H_0 (b) $z = 2.206$, accept H_0
   (c) $z = -1.79$, accept H_0 (d) $z = 2.118$, accept H_0
   (e) $z = -2.937$, reject H_0
2. $z = -2.40$, do not accept claim as there is evidence that proportion is less.
3. $z = 1.637$, yes
4. $z = 1.476$, no
5. $z = -1.990$, no
6. $z = 1.5$, no
7. $z = 1.705$, evidence that more than 65% own a mobile phone.
8. (a) (i) 0.0297 (ii) 0.0934
   (b) $z = -1.792$, germination rate less than 75% (only just – do further tests)
9. $z = 2.43$, yes
10. Replies were representative of the population.
    (a) $z = 1.220$, no evidence to suggest proportion in favour is more than 0.7.
    (b) (0.681, 0.808)
11. (a) Evidence that proportion is lower
    (b) No different
12. (a) $z = -3.03$, evidence that $p < 0.4$
    (b) (0.379, 0.458); 75
13. $z = -1.267$, no
14. $z = -2.44$, evidence that proportion has fallen

Exercise 11d testing the difference between means of two normal populations

Section A: z-tests (page 543)

1. (a) (i) $z = -2.096$, reject H_0 (ii) $z = -1.402$, accept H_0
   (iii) $z = 2.493$, reject H_0
   (b) (i) $z = 1.99$, accept H_0 (ii) $z = 2.076$, reject H_0
   (iii) $z = -2.036$, accept H_0 (iv) $z = 1.783$, reject H_0
   (v) $z = 1.779$, reject H_0 (vi) $z = -2.321$, accept H_0 (only just) (vii) $z = 2.55$, reject H_0
2. 0.567, $z = -2.219$, flowers on sunny side grow taller
3. $z = 3.52$, second population has smaller mean than first
4. $z = 2.036$, significant at 5% level, not significant at 4% level
5. 4.41, (9.87, 10.73), 3.61, $z = 1.49$, not significant evidence
6. $z = -1.646$, reject Mr Brown's claim (only just)
7. $z = -2.04$, evidence of difference
8. 1.15, $z = -2.913$, significant evidence
9. $z = 1.627$, accept; 124
10. 27.33 (26.77, 27.89), 2.4, $z = 1.97$, those of higher intelligence do not have greater foot length.

Section B: t-tests (page 545)

1. (i) (a) 17.73 (b) $t = 2.135$, reject H_0
   (ii) (a) 87.09 (b) $t = -0.567$, accept H_0
   (iii) (a) 27.5625 (b) $t = 2.088$, accept H_0
   (iv) (a) 4.182 (b) $t = 1.260$, accept H_0
2. $t = -1.13$, no evidence that Welsh policemen are shorter than Scottish policemen.
3. 196, $z = -1.714$, do not differ significantly
4. (a) 10.8125 (b) $t = -1.282$, accept claim
5. $t = 2.423$, not significant difference
6. Normal populations with common variance, $t = 2.36$, evidence that mean has increased; $t = 2.041$, the mean could be 500 g.
7. (a) Normal populations with common variance
   (b) $H_0: \mu_1 = \mu_2$, $H_1: \mu_1 \neq \mu_2$ (c) $t = -0.942$, same
8. $t = 1.868$, evidence that new method has led to higher scores; (−2.60, 33.9)

Miscellaneous exercise 11e (page 554)

1 (17.1, 19.7), there is a 10% chance that it hasn't trapped μ: $z = 0.759$, μ could be 17.8
2. (a) Children within families selected are representative of all children.
   (b) $z = 0.939$, data do not indicate that boys and girls are not equally likely
3. (a) $H_0: \mu = 30$, $H_1: \mu > 30$ (b) $\bar{x} > 33.95$
   (c) Evidence that mean speed is greater than 39 mph ($\bar{x}$ is in critical region).
   (d) 0.9941
4. (a) $H_0: \mu = 43$, $H_1: \mu > 43$
   (b) Since n is large, the distribution of the sample means is approximately normal
   (c) $z = 1.768$, mean amount has increased
   (d) (43.35, 52.65), consistent, 43 out of range of confidence interval
5. $H_0: p = 0.13$, $H_1: p < 0.13$, 2%, 0.161
6. (a) (i) $P(\bar{X} >$ critical value $| \mu = 65)$
   (ii) $P(\bar{X} <$ critical value $| \mu$ is value specified by the alternative hypothesis)
   (c) Accept H_0, Type II
   (d) 0.0059, Type II error would be less and tends to zero as μ increases
7. Not representative as it excludes people at work, school, etc; better to take random samples at random times during the day for a spread of days, (68%, 80%). $z = -2.03$, data provide significant evidence
8. (a) $\bar{x} > \mu_0 + 1.96\frac{\sigma}{\sqrt{n}}$ or $\bar{x} < \mu_0 - 1.96\frac{\sigma}{\sqrt{n}}$
   (b) $\bar{x} \geqslant \mu_0 - 2.326\frac{\sigma}{\sqrt{n}}$
9. (a) 0.422
   (b) E(unbiased estimate) = true value; batch not rejected, 9.6%
10. (a) $H_1: p > \frac{1}{6}$ (b) $N \geqslant 50$ (c) $0.059 \approx 6\%$
11. Accept as slow if mean bounce <11.645, 0.0004
12. (a) (i) 10.46 (ii) 15.64, E(unbiased estimate) = true value
    (b) 1
    (c) Central Limit theorem holds when n is large
13. $z = 3.367$, accept claim that mean duration is more than 12 months; n large, use Central Limit theorem
14. (a) 75
    (b) $z = 2.19$, machine is not correctly calibrated
    (c) Unbiased estimate of standard deviation used, distribution of sample mean approximately normal; (0.316, 5.684)
    (d) Smaller, might lead to result that machine is correctly calibrated.
15. (a) 66.25, 133.40 (b) $H_0: \mu = 62.5$, $H_1 > 62.5$, $z = 1.465$, no evidence of increase
16. (a) $z = 2.475$, mean has increased

17. (a) $H_0: \mu = 1.73$, $H_1: \mu > 1.73$ (b) $\bar{X} \sim N(1.73, 0.0008)$
    (c) $\bar{x} > 1.777$
    (d) men who play basketball are not taller (e) 0.14
18. 9

Test 11A (z-tests) (page 558)

1. (a) 21.25
   (b) $z = 0.99$, no evidence to support manufacturer's suspicion
   (c) obtaining distribution of $\bar{X}$, distribution of X not known
2. $z = 1.567$, not sufficient evidence to say that the quoted figure is an underestimate
3. (a) (i) P(reject H_0 when H_0 is true)
   (ii) P(accept H_0 when H_1 is true)
   (b) (i) $z = 2.372$, mean is greater than 17.5
   (ii) $\bar{x} > 18.09$
   (iii) 0.639
4. 0.0606 (≈6%), 0.1118

Test 11B (z-tests) (page 558)

1. $H_0: \mu = 125$, $H_1: \mu < 125$, $z = -1.549$, no evidence that μ is lower
2. $z = 2.318$, government spokesman
3. (a) $\bar{x} < 59.82$
   (b) It is accepted that the mean is 60 when in fact it is an alternative value (less than 60).
   (c) 0.057
4. (a) $H_0: \mu_1 - \mu_2 = 0$, $H_1: \mu_1 - \mu_2 \neq 0$
   (b) $z = 1.6$, no difference
   (c) Distribution likely to be skewed rather than symmetric

Test 11C (t-tests) (page 559)

1. $t = -3.560$, evidence that mean falls below $7.40; normal distribution
2. $t = -2.915$, San Marco cooler
3. (a) 4.238 (b) Normal distribution, $t = 2.857$, yes
   (c) Perform z-test not t-test
4. $t = -2.046$, new score higher; (−6.948, 32.282) or (−32.282, 6.948)

Chapter 12

There will be variation in answers, depending on the degree of accuracy used in various stages of the working.

Exercise 12a Goodness of fit test – uniform and given ratio (page 569)

1. $X^2 = 1.93$, $\nu = 3$, die is fair
2. $X^2 = 18.16$, $\nu = 9$, uniform distribution
3. $X^2 = 6.19$, $\nu = 2$, yes
4. $X^2 = 4.95$, $\nu = 3$, no; $X^2 = 9.90$, $\nu = 3$, yes
5. $X^2 = 8.24$, $\nu = 7$, accept theory
6. $X^2 = 4.15$, $\nu = 4$, yes
7. $X^2 = 10.68$, $\nu = 4$, no
8. 15.5
9. 78.81, 17.8, 7.8, 6.7 $X^2 = 5.92$, $\nu = 3$, no difference
10. $X^2 = 38.2$, $\nu = 9$, evidence of bias
11. $X^2 = 10$, $\nu = 4$, not uniform
12. $X^2 = 4.4$, $\nu = 5$, die is fair
13. (a) modal class 2 to <4
    (b) 4 years 8 months, 3 years 2 months
    (c) For cumulative frequency curve plot (0, 0), (2, 42), (4, 94), (6, 122), (8, 142), (10, 160), (12, 176); 3 years 9 months, 4 years 9 months
    (d) $X^2 = 5.73$, $\nu = 5$, justified

Exercise 12b Goodness of fit tests – binomial, Poisson and normal distributions (page 579)

1. Combine last three classes, $X^2 = 4.09$, $\nu = 3$, accept $X \sim B(5, 0.3)$
2. $X \sim B(5, \frac{1}{6})$, E = 80.5, 80.5, 32, 7 (last 3 classes combined), $X^2 = 8.21$, $\nu = 3$, biased; $\bar{x} = 1$, $p = 0.2$, $X \sim B(5, 0.2)$, E = 66, 82, 41, 11 (last three classes combined) X^2 is very small, $\nu = 2$, too good a fit, query data.
3. np, 1.6, 0.32, E = 7.3, 17.1, 16.1, 7.5, 1.8, 0.2 (combine last 3 classes) $X^2 = 1.79$, $\nu = 2$, good fit
4. $X \sim B(2, \frac{1}{6})$, E = 150, 60, 6, $X^2 = 9.6$, $\nu = 2$, reject; use $\bar{x} = 0.444$, $p = 0.222$, find E, $\nu = 1$
5. (a) $\bar{x} = 2$, $p = 0.4$, E = 6, 21, 28, 18, 7 (combine last 2 classes)
   (b) $X^2 = 2.21$, $\nu = 3$ (c) yes, binomial adequate
6. (a) $X \sim B(5, 0.2088)$, E = 155, 205, 108, 28, 4, 0 (combine last 3 classes)
   (b) $X^2 = 5.959$, $\nu = 2$, binomial (but only just)
7. (a) 7
   (b) $n = 20$, $p = 0.35$; 0.16135, 8.1
   (c) 12.3
   (d) E = 12.3, 8.6, 9.2, 8.1, 11.8, O = 9, 7, 17, 8, 9, $X^2 = 8.46$ (e) 3, not good fit at 5% level
8. (a) E = 246.6, 345.2, 241.7, 112.8, 39.5, 14.2
   (b) $X^2 = 32.2$, $\nu = 5$, not accepted
9. $\bar{x} = 2.5$, E = 8, 21, 26, 21, 13, 11 (combine end classes), $X^2 = 2.59$, $\nu = 4$, good
10. $\bar{x} = 1.28$, E = 41, 52, 34, 14, 6 (combine end classes), $X^2 = 6.81$, $\nu = 3$, not significant
11. $\bar{x} = 0.65$, E = 20.88, 13.57, 5.55 (combine end classes), $X^2 = 1.85$, $\nu = 1$, accept
12. (a) $\bar{x} = 1.2$, E = 99, 119, 72, 29, 9, 2 (combine end classes)
    (b) $X^2 = 0.48$, $\nu = 3$, very good fit
13. $\bar{x} = 0.9$, E = 21, 18, 11 (combine last 3 classes), $X^2 = 1.80$, $\nu = 1$, yes, consistent
14. (b) E = 7.3, 12.4, 10.6, 9.7
    (c) $X^2 = 177$, $\nu = 2$, reasonable
    (d) very low, suspicious
15. E = 6.68, 9.19, 14.98, 19.15, 19.15, 14.98, 9.19, 6.68, $X^2 = 3.197$, $\nu = 7$, accept.
    If μ, σ^2 unknown, $\nu = 5$
16. (a) E = 3, 13, 28, 32, 18, 6 (combine first 2 classes), $X^2 = 11.9$, $\nu = 4$, reject
    (b) $\bar{x} = 171.54$, $s = 7.11$, E = 6, 18, 32, 28, 13, 3 (combine last 2 classes), $X^2 = 1.73$, $\nu = 2$, accept normal
17. (a) $\bar{x} = 1.732$, $\hat{\sigma} = 0.216$ (3 d.p.), E = 7.78, 26.05, 44.12, 33.64, 13.41, $X^2 = 8.96$, $\nu = 2$
    (b) $X^2 = 2.42$, $\nu = 1$

Exercise 12c Contingency tables (page 588)

1. E = 48, 10.67, 21.33, 42, 9.33, 18.67, $X^2 = 1.037$, $\nu = 2$, no difference
2. E = 25.5, 25.5, 60.5, 60.5, 26.5, 26.5, 7.5, 7.5, $X^2 = 2.03$, $\nu = 3$, independent
3. E = 50.1, 29.5, 23.4, 22.9, 13.5, 10.6, $X^2 = 4.00$, $\nu = 2$, yes
4. E = 65.1, 28.9, 58.9, 26.1, $X^2 = 7.43$, $\nu = 1$, yes
5. E = 27.5, 972.5, 27.5, 972.5, $X^2 = 4.79$, $\nu = 1$, yes

6. E = 11.4, 14.3, 8.6, 15.7, 18.3, 22.9, 13.7, 25.1, 20.6, 25.7, 15.4, 28.3, 29.7, 37.1, 22.3, 40.9, $X^2 = 12.0$, $\nu = 9$, accept
7. E = 66.7, 33.3, 53.3, 26.7, $X^2 = 6.81$, $\nu = 1$, no
8. E = 21.0, 10.0, 7.0, 15.5, 7.5, 5, 41.5, 19.5, 14, $X^2 = 7.86$, $\nu = 4$, accept
9. E = 34.2, 29.8, 12.8, 11.2, $X^2 = 1.22$, $\nu = 1$, no
10. E = 17.5, 82.5, 17.5, 82.5, $X^2 = 0.58$, $\nu = 1$, no
11. E = 202.2, 260.7, 318.1, 184.8, 238.3, 290.9, $X^2 = 2.02$, $\nu = 2$, independent
12. $\nu = 2$, $X^2 = 5.99$
13. (a) $\nu = (3 - 1)(3 - 1) = 4$ (b) difference
14. E = 13.5, 15.5, 21, 8.64, 9.92, 13.44, 31.86, 36.58, 49.56, $X^2 = 11.35$, $\nu = 4$, yes
15. E = 33.6, 22.4, 63.6, 42.4, 22.8, 15.2, $X^2 = 4.775$, $\nu = 2$, no difference
16. E = 90.405, 56.595, 35.595, 20.405, $X^2 = 13.3$, $\nu = 1$, related

Miscellaneous exercise 12d (page 594)

1. H_0: Preference for proposed route is independent of where people live (no association between them) H_1: There is an association between them E = 47, 28, 31.33, 18.67, 15.67, 9.33, $X^2 = 1.479$, $\nu = 2$, no association
2. (a) H_0: Occurrence of shoplifting is uniformly distributed between the months, H_1: Shoplifting is more likely to occur in some months than others.
   E = 14.5 (all classes), $X^2 = 14.268$, $\nu = 11$, no association
3. H_0: No association between reaction and eye colour, H_1: There is an association between them, E = 15.675, 7.425, 9.9, 26.125, 12.375, 16.5, 15.2, 7.2, 9.6, $X^2 = 20.9$, $\nu = 4$, association between reaction and eye colour
4. E = 6.3 (combines first 4 classes), 8.85, 14.66, 18.99, 19.28, 15.32, 9.52, 7.05 (combine last 3 classes), $X^2 = 4.908$, $\nu = 6$, good fit
5. (a) H_0: No association between brand of fertiliser and yield, H_1: There is an association between them, E = 10, 12, 8, 8, 9.6, 6.4, 7, 8.4, 5.6, $X^2 = 7.811$, $\nu = 4$, no association
   (b) $\nu = 2$, there is an association between choice of company and yield
   (c) Quickgrow
6. (a) H_0: Peak flow measurements are normally distributed (with mean and variance as estimated from the data), accept
   (b) Expected frequency must be greater than 5, combine classes
7. (a) E = 18.33, 20.67, 20.68, 23.32, 7.99, 9.01
   (b) $X^2 = 6.88$, $\nu = 2$, there is an association
8. (a) H_0: No association between candidates' grades in mathematics and physics, H_1: There is an association between them, E = 17.2, 13.8, 12.8, 10.2, $X^2 = 5.672$, $\nu = 1$, there is an association
   (b) Expected frequency might drop below 5
9. (a) E = 44.62, 66.94, 50.2, 25.12, 13.12 (combine last two), $X^2 = 10.6$, $\nu = 4$, at 5% level, no
   (b) Use mean from data for λ, $\nu = n - 2 = 3$
   (c) Do not have independent events with a constant probability of success.
10. E = 644 (all classes), $X^2 = 10.95$, $\nu = 4$, data do not support claim, random events, $\bar{x} = 1.1$, E = 33.3, 36.6, 20.1, 10 (combining last 2 classes), $X^2 = 0.48$, $\nu = 2$, accept
11. (a) H_0: Grades are in the ratio 15 : 20 : 35 : 25 : 5, H_1: Grades are not in this ratio E = 30, 40, 70, 50, 10, $X^2 = 7.074$, $\nu = 4$, same proportion
    (b) H_0: Sex and grade are not associated, H_1: There is an association between them, $\nu = 4$, there is an association
12. (a) $\bar{x} = 0.74$, combine 4 and over, E = 667.96, 494.29, 182.89, 45.11, 9.75, $X^2 = 108.87$, $\nu = 3$, not adequate
    (b) Not consistent, Poisson model was not adequate
    (c) E = 259 (all classes), $X^2 = 13.8$, $\nu = 3$, not consistent
13. (a) E = 19.33, 15.33, 10, 15.33, 9.67, 7.67, 5, 7.67, $X^2 = 12.08$, $\nu = 3$, mark is associated with type of question
    (b) Poisson, this is most similar question
    (c) E = 22.5, 22.5, 22.5, 22.5, 7.5, 7.5, 7.5, 7.5, $X^2 = 17.6$, $\nu = 3$, yes it is
    (d) Contingency table – popular and well answered; Binomial and Poisson fits – average popularity, relatively badly answered, normal fit – unpopular but well answered by those who attempted it.
14. (a) E = 480 (all classes), $X^2 = 14.8$, $\nu = 4$, there is evidence
    (b) E = 6.405, 6.51, 8.085, 24.705, 25.11, 31.185, 29.89, 30.38, 37.73, $X^2 = 16.9$, $\nu = 4$, length of employment is associated with grade
15. E = 38.78, 34.02, 18.2, 39.21, 34.39, 18.4, 27.28, 23.92, 12.8, 22.59, 19.81, 10.6, 22.59, 19.81, 10.6, 28.55, 25.05, 13.4, $X^2 = 16.0$, $\nu = 10$, no association, expected frequency must be greater than 5, would not make sense; E = 60(all classes), $X^2 = 13.2$, $\nu = 6$, reject hypothesis
16. (a) $\bar{x} = 1$, E = 36.79, 36.79, 18.39, 8.026, (3 or more), $X^2 = 12.9$, $\nu = 2$, not suitable
    (b) E = 26.25, 6.75, 8.75, 2.25, $X^2 = 3.77$, $\nu = 1$, yes

Mixed test 12A (page 598)

1. (a) 1.04 (b) Calls occur at random
   (c) E = 58.01, 55.11, 26.18, 10.70, $X^2 = 4.86$, $\nu = 3$, Po(0.95) is suitable
2. E = 6, 20, 12, 2, 9, 30, 18, 3, $X^2 = 13.11$, $\nu = 3$, there is a link between General Studies performance and degree class
3. E = 15.9, 21.1, 26.1, 21.1, 15.9 (combine first 2 and last 2 classes), $X^2 = 7.08$, $\nu = 4$, N(180, 9) is suitable
4. H_0: There is no association between gender and passing a driving test.
   H_1: There is an association.
   E = 27.5, 22.5, 27.5, 22.5, $X^2 = 2.585$, $\nu = 1$, results do not indicate link

Mixed test 12B (page 599)

1. E = 32 (all classes), $X^2 = 1.6875$, $\nu = 3$, no particular preference, data cannot be used to discredit claim
2. E = 21.34, 43.66, 21.66, 44.34, $X^2 = 4.72$, $\nu = 1$, there is an association between the two factors
3. E = 16.67, 16.67, 16.67, 16.67, 33.33, 50, $X^2 = 4.65$, $\nu = 5$, die is biased in the way described
4. (b) Random positions (c) 37.24, 2.50
   (d) H_0: The distribution can be modelled by Po(2.59), H_1: The distribution cannot be modelled by Po(2.59), $X^2 = 7.55$, $\nu = 5$, Poisson model is supported by data

Chapter 13

Exercise 13a Significance test for product – moment correlation coefficient (page 604)

1. Reject a, c, f, g, h: do not reject b, d, e
2. (a) 0.3755
   (b) H_0: $\rho = 0$, H_1: $\rho > 0$, reject H_0, no evidence
   (c) X and T are jointly normally distributed with correlation coefficient ρ and that the data constitute a random sample from all values of x and t.
3. (a) Scatter diagram
   (b) 0.834
   (c) reasonable
   (d) Student's view is wrong; correlation does not imply causation; in this case there may be a common underlying cause such as wealth.
4. (a) −0.690, reject H_0 in favour of H_1
   (b) 0.686, reject H_0 in favour of H_1

Exercise 13b Significance test for Spearman's rank correlation coefficient (page 607)

1. Reject b, f, h; do not reject a, c, d, g, i, e
2. 0.714, no evidence of agreement (only just)
3. (a) 0.52
   (b) H_0: $\rho_s = 0$, H_1: $\rho_s > 0$, do not reject H_0, no evidence of agreement between the interviewers
4. 0.745, evidence of correlation
5. (a) 0.66
   (b) evidence of positive correlation
6. 0.4286, H_0: $\rho_s = 0$; no evidence of positive correlation

Miscellaneous exercise 13c (page 612)

1. (a) 0.636
   (b) H_0: $\rho_s = 0$, H_1: $\rho_s > 0$. Accept H_0, no evidence of positive correlation.
2. (b) 0.916
   (c) Evidence of positive correlation between the number of wren territories recorded and the number of adult wrens trapped.
3. (b) −0.975 (c) strong negative correlation
   (d) There is evidence of correlation between hours of sunshine and temperature.
4. (a) 0.4286
   (b) H_0: $\rho_s = 0$, H_1: $\rho_s \neq 0$, no evidence of correlation between attendance and position in the league
5. 0.527, no evidence of agreement
6. (b) 0.535
   (c) some positive correlation
   (d) Low mark in x, high mark in y
   (e) 0.794
   (f) H_0: $\rho_s = 0$, H_1: $\rho_s > 0$, evidence of positive correlation
7. (a) $c = 2.48 + 0.607m$
   (b) 0.593
   (c) 11.4 (d) 0.516, no (e) r
8. 0.690, H_0: $\rho_s = 0$, H_1: $\rho_s \neq 0$, no evidence of correlation
9. (b) 0.783
   (c) H_0: $\rho = 0$, H_1: $\rho > 0$, evidence of positive correlation
   (c) Data constitute a random sample of all values of x and y, years selected may no be representative.
   (d) lower
10. 0.825, 0.929, evidence of positive correlation (1% level)
11. −0.3341, not significant (5%), −0.6939, significant (2.5%)

Mixed test 13A Correlation coefficients (page 615)

1. (a) 0.473 (b) evidence
2. (a) 0.667
   (b) H_0: $\rho_s = 0$, H_1: $\rho_s > 0$, judges in broad overall agreement
   (c) evidence of correlation
   (d) Spearman's rank
3. $r = 0.310$, not significant, no evidence of possible correlation
4. (a) 0.619
   (b) H_0: $\rho_s = 0$, H_1: $\rho_s > 0$, not evidence of positive correlation (just)
   (c) Two very different sets of data being compared